A DNS study of Momentum and Energy Exchange in Misaligned Wind-Wave Condition

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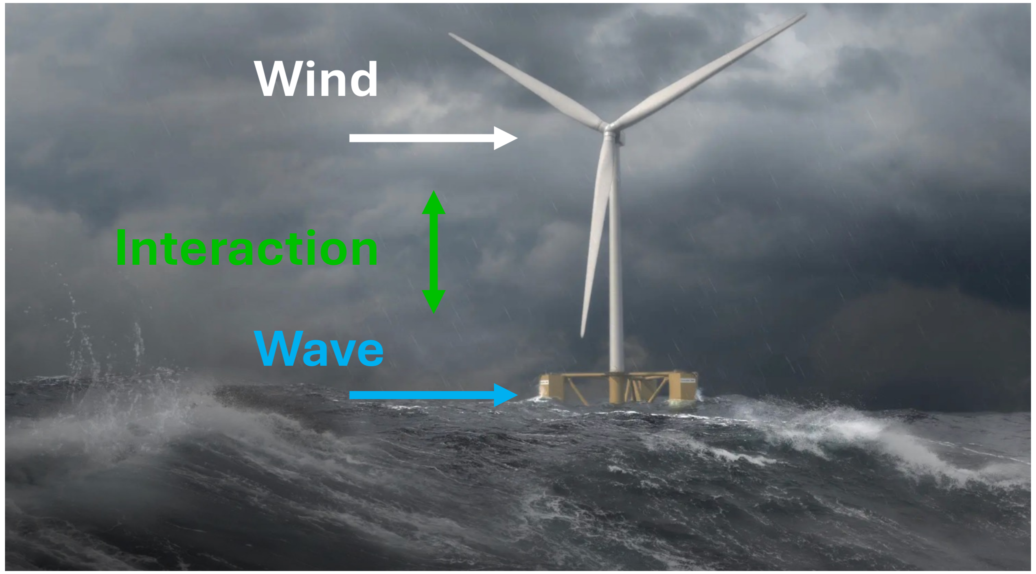
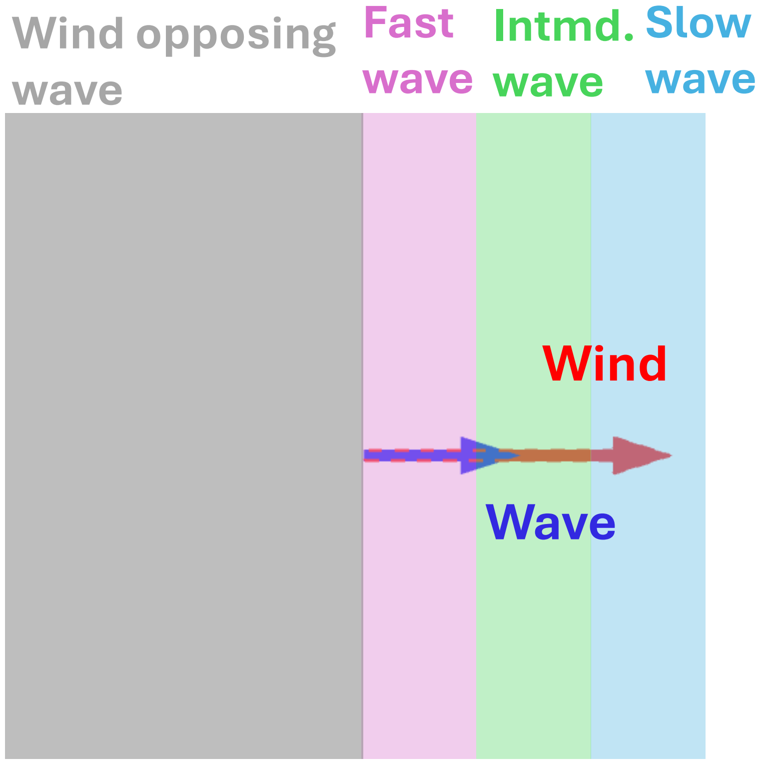
- Wind-wave interaction
  - Exchanges momentum/energy
  - Promotes wave growth/decay
  - Modifies the mean wind profile
- Understanding wind-wave interaction is crucial for
  - Predicting wave field
  - Optimizing offshore wind turbine efficiency



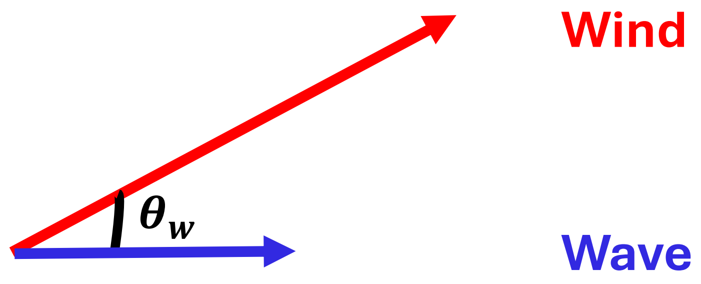
Source: NOV.com

- Misaligned wind-waves
  - Common in storms (slow wave) & coastal swells (fast wave)
  - Distinct interaction mechanism

Align-wave component of wind



- Misaligned wind-waves: previous parametrization
  - Mostly extended from aligned wind-wave studies via streamwise projection.



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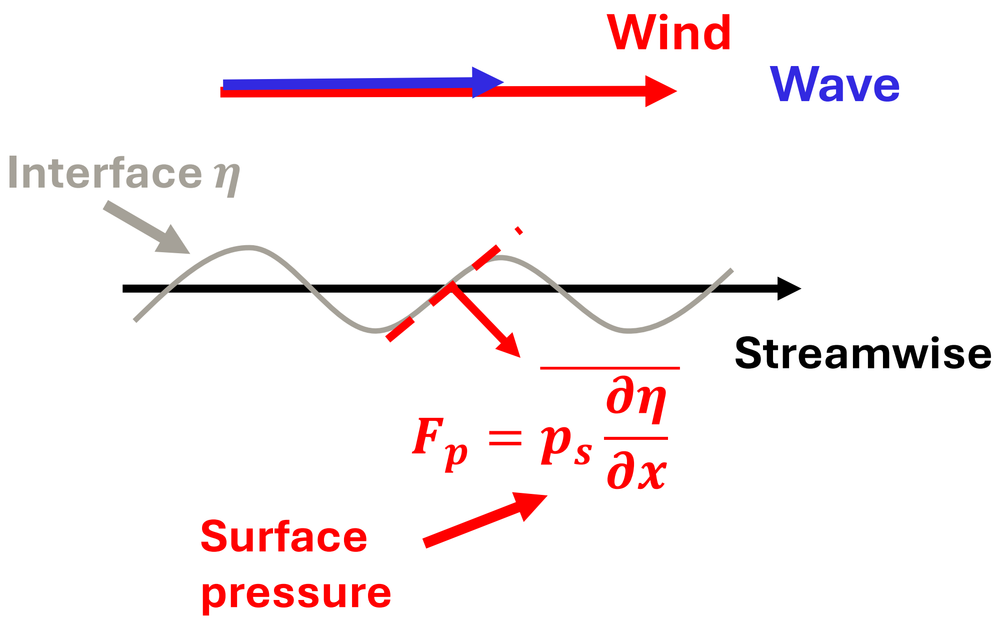
- **Form drag:** $F_p \equiv \overline{p_s \frac{\partial \eta}{\partial x}}$
  - Jansen 1991: $F_p \propto u_*^2$

Frictional velocity



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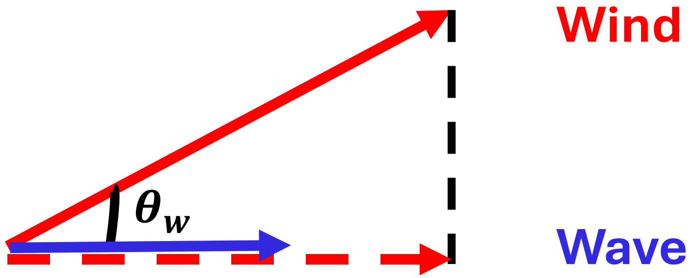

- **Form drag:** $F_p \equiv \overline{p_s \frac{\partial \eta}{\partial x}}$
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  - Donelan 2012: $\boldsymbol{F_p} \propto (\frac{\mathrm{U}(\lambda/2)}{\mathrm{c}} - 1)^2/\mathrm{c}$

Wavelength

Wave sheltering

- Misaligned wind-waves: previous parametrization
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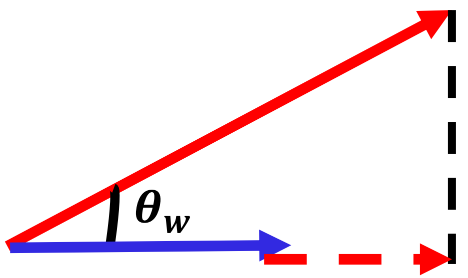

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- Misaligned wind-waves: previous numerical studies
  - Mostly are one phase (air) models with prescribed wave movement (e.g., Husain 2022 (LES), Deskos 2022 (DNS))

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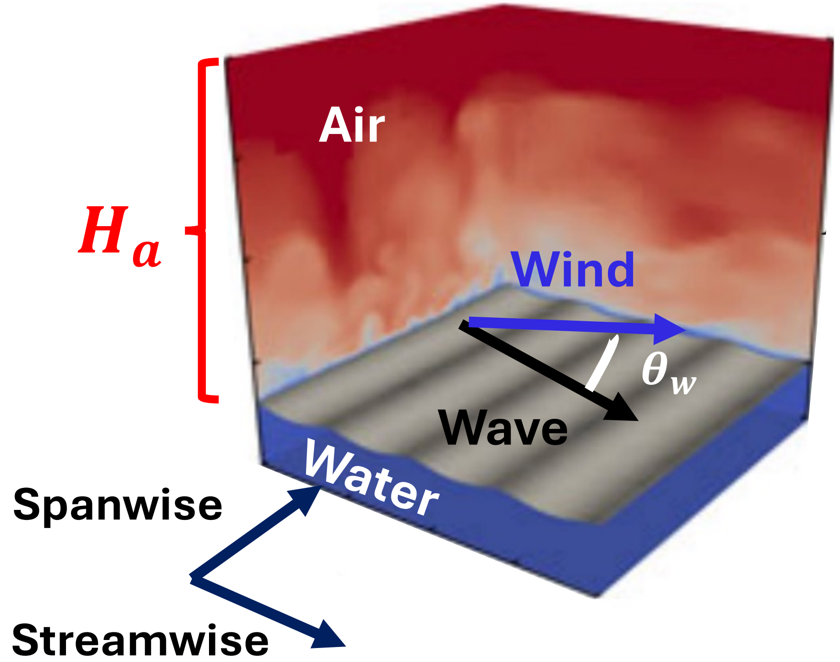
Propose

Use two-phase fully coupled DNS model (via Basilisk solvers) without any assumptions/parametrizations (e.g., Wu 2022)

- Setup
  - Pressure driven airflow with given $Re_* = \frac{u_* H_a}{\nu_a}$ and θ_w

    Airflow height (H_a); Airflow viscosity (ν_a); Misalignment angle (θ_w)
  - Stokes wave profile with given a and k

    Amplitude (a); Wavenumber (k)



Wu et al. (2022)

- Parameter space

  - Wave steepness: $ak = [0.1, 0.2]$;

  - Misalignment angle: $\theta_w = \left[0 \colon \frac{\pi}{12} \colon \pi\right]$ rad;

    Wind following ← → **Wind opposing**

  - Inverse wave age: $u_*/c = [0.125, 0.25, 0.5]$;

    Fast wave ← → **Slow wave**

- Parameter space

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    Wind following ⟵ ⟶ **Wind opposing**

  - Inverse wave age: $u_*/c = [0.125, 0.25, 0.5]$;

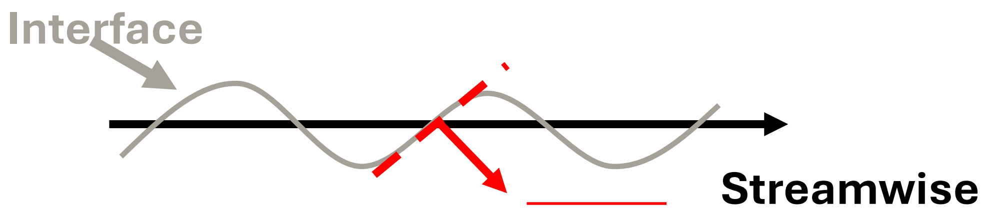
    Fast wave ⟵ ⟶ **Slow wave**

Output

$\vec{u}$, p & η
in (x, y, z, t)

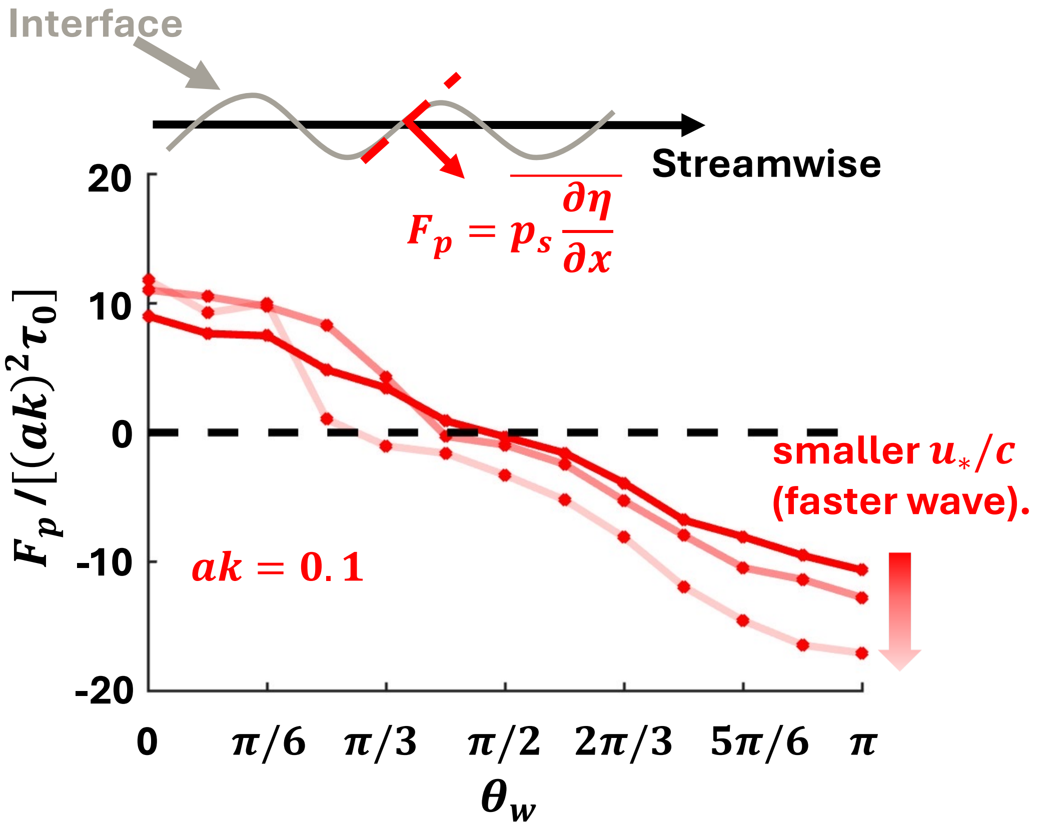
- F_p: Form drag
- $S_{\mathrm{in}} \approx F_p c$: Pressure input
- Surface shear direction

- F_p: Form drag



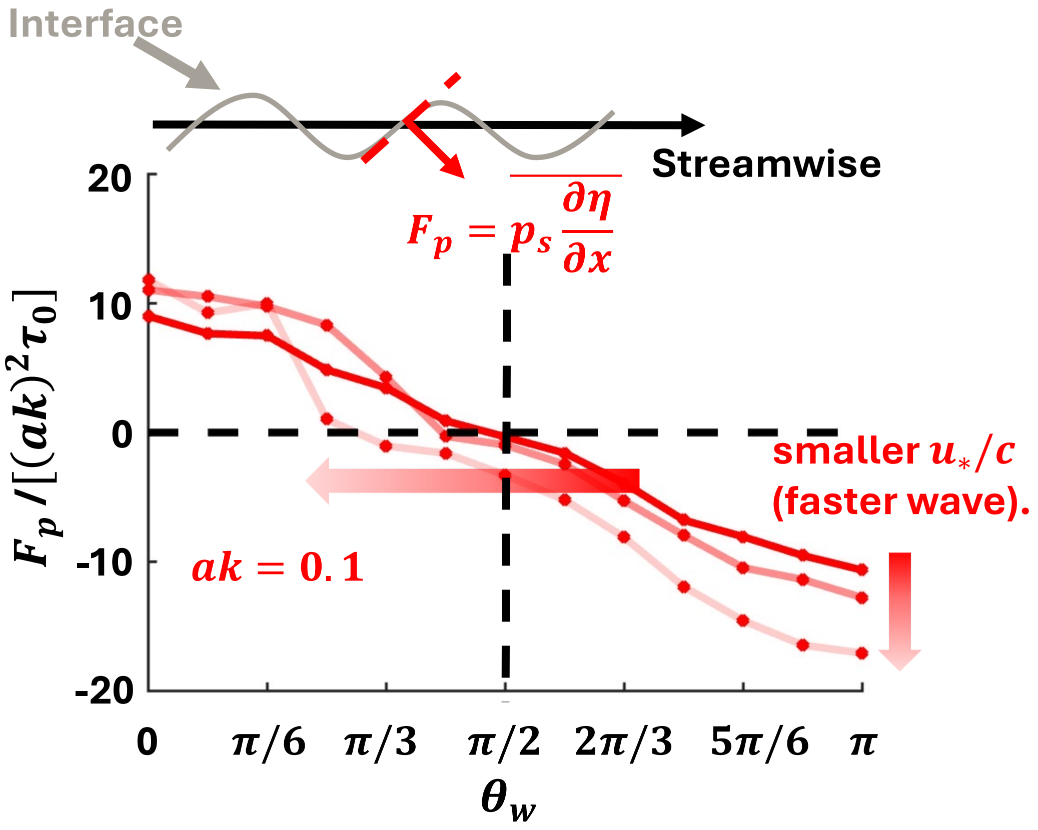
$$F_p = \overline{p_s \frac{\partial \eta}{\partial x}}$$

- F_p: Form drag

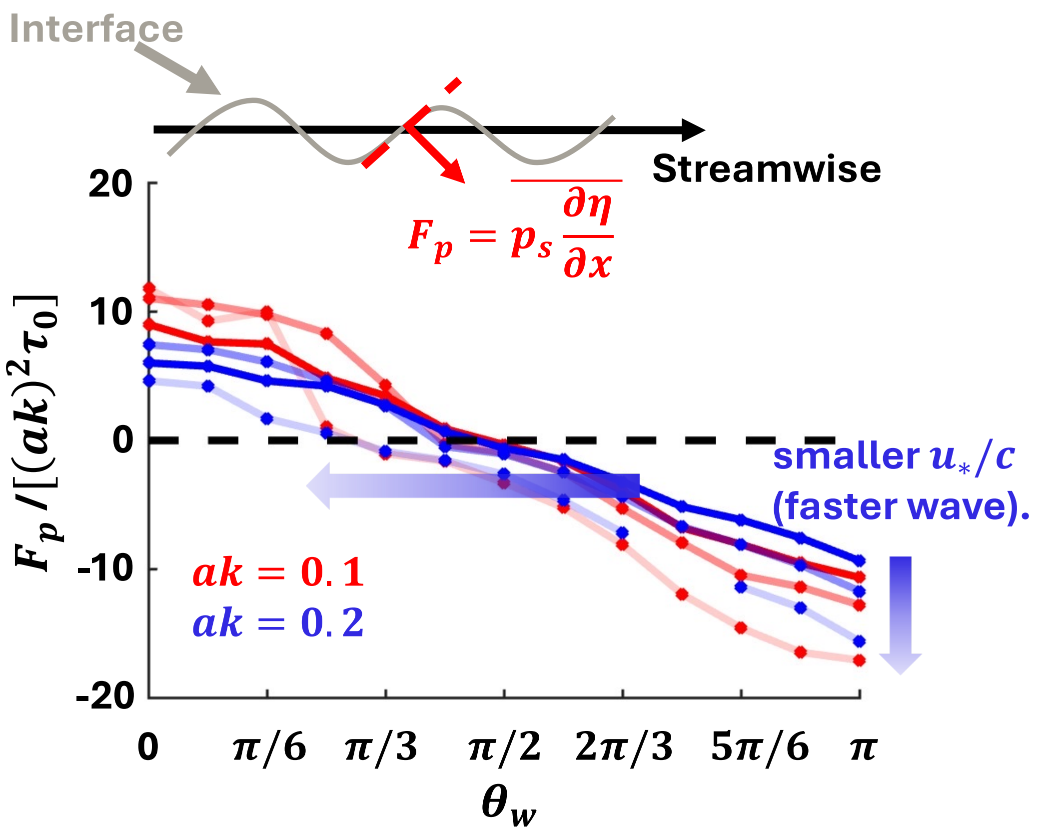

Interface
Streamwise
$F_p = p_s \frac{\partial \eta}{\partial x}$
20
10
0
-10
-20
$F_p/[(ak)^2\tau_0]$
$ak = 0.1$
smaller u_*/c
(faster wave).
0
$\pi/6$
$\pi/3$
$\pi/2$
$2\pi/3$
$5\pi/6$
π
θ_w

- F_p: Form drag
  - Asymmetric
  - Zero-crossing at smaller θ_w with smaller u_*/c (faster wave).
  - Do not follow $\cos^2(\theta_w)$

May be related to the relative streamwise wind velocity



- F_p: Form drag
  - Asymmetric
  - Zero-crossing at smaller θ_w with smaller u_*/c (faster wave).
  - Scales with $(ak)^2$

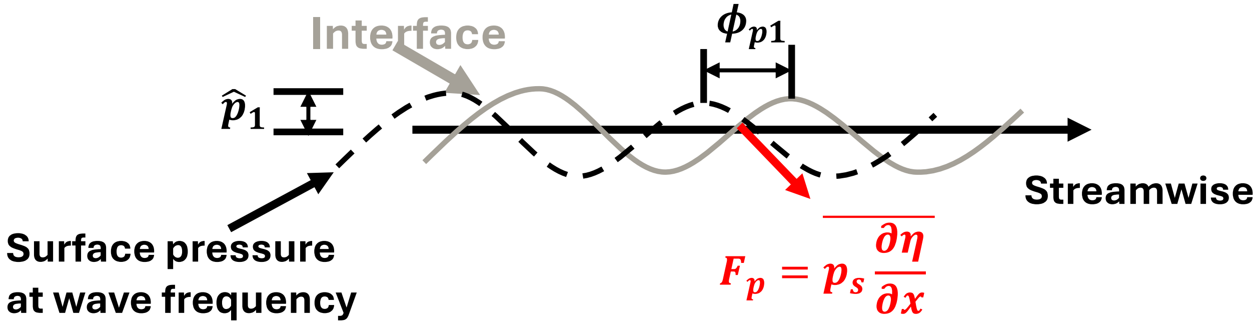


- F_p: Form drag

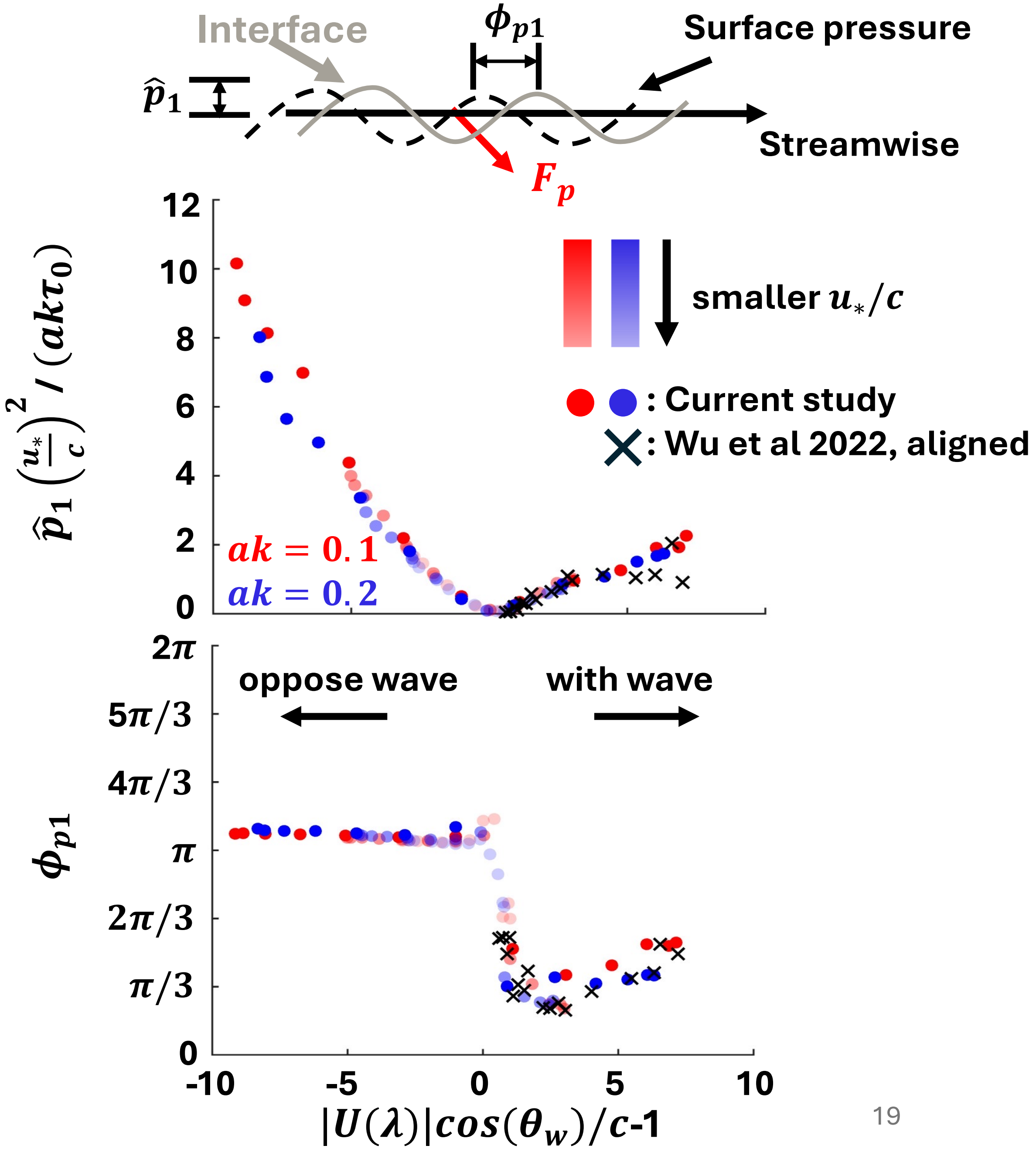
Phase lag

$$F_p = \frac{1}{2} ak\hat{p}_1 \sin(\phi_{p1})$$

Pressure amplitude



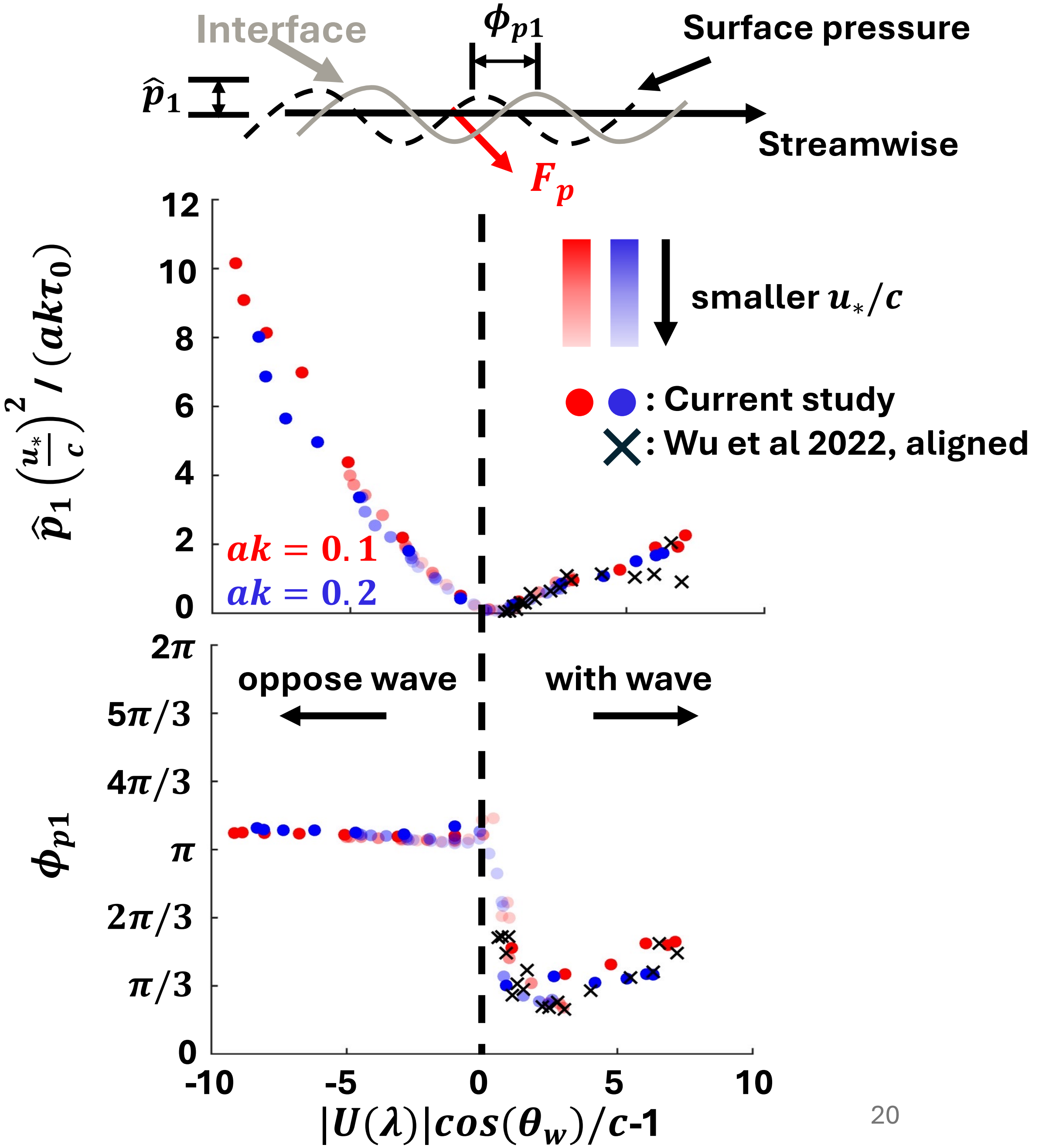
- F_p: Form drag


Interface
ϕ_{p1}
Surface pressure
$\widehat{p}_1$
Streamwise
F_p
smaller u_*/c
: Current study
: Wu et al 2022, aligned
$ak = 0.1$
$ak = 0.2$
$\widehat{p}_1\left(\frac{u_*}{c}\right)^2/(ak\tau_0)$
oppose wave
with wave
ϕ_{p1}
$|U(\lambda)|cos(\theta_w)/c$-1

- F_p: Form drag

  $\widehat{p}_1$: Pressure amplitude

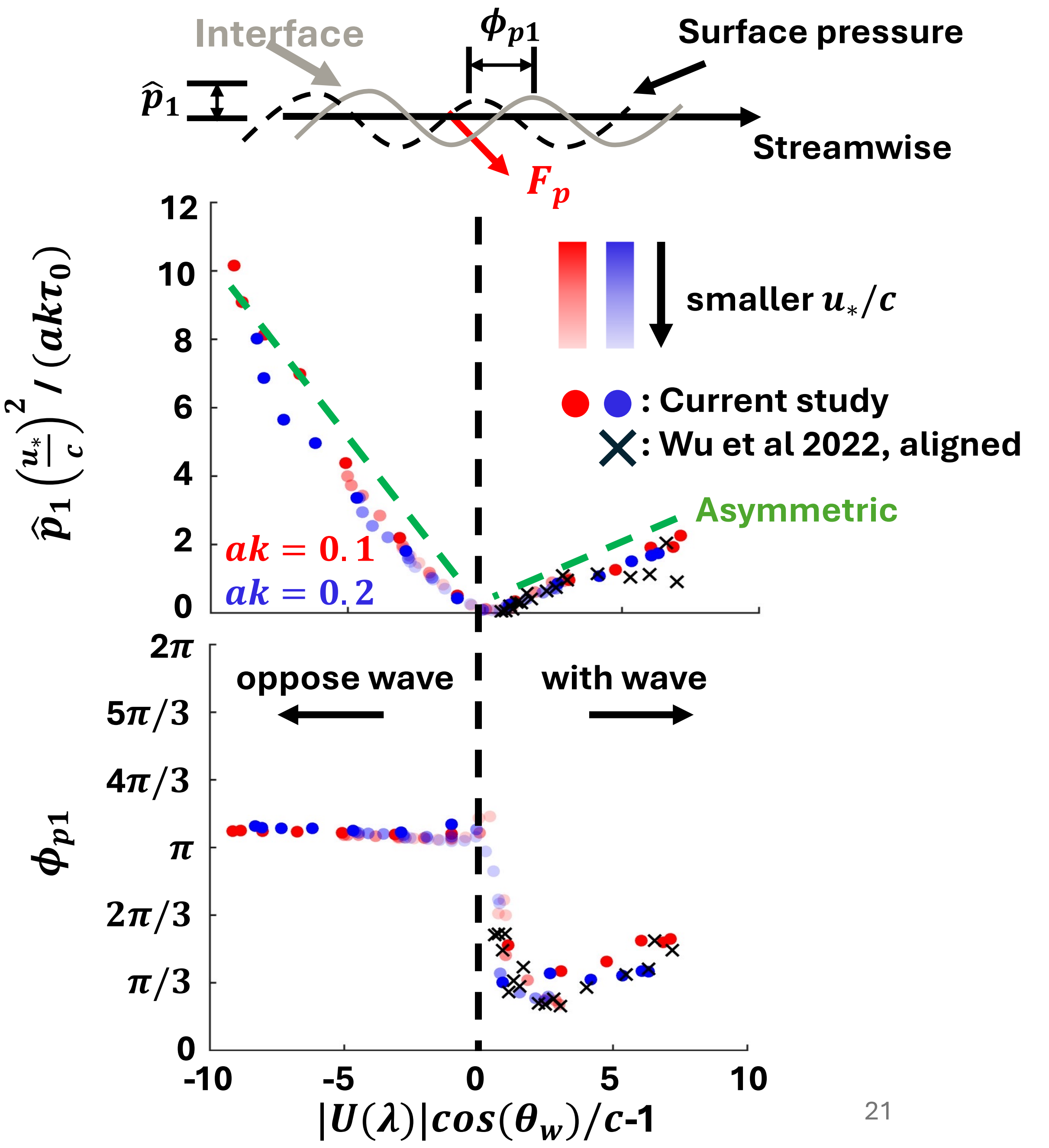
  - Minimum when $|U(\lambda)|\cos(\theta_w) \approx c$



- F_p: Form drag

  $\hat{p}_1$: **Pressure amplitude**

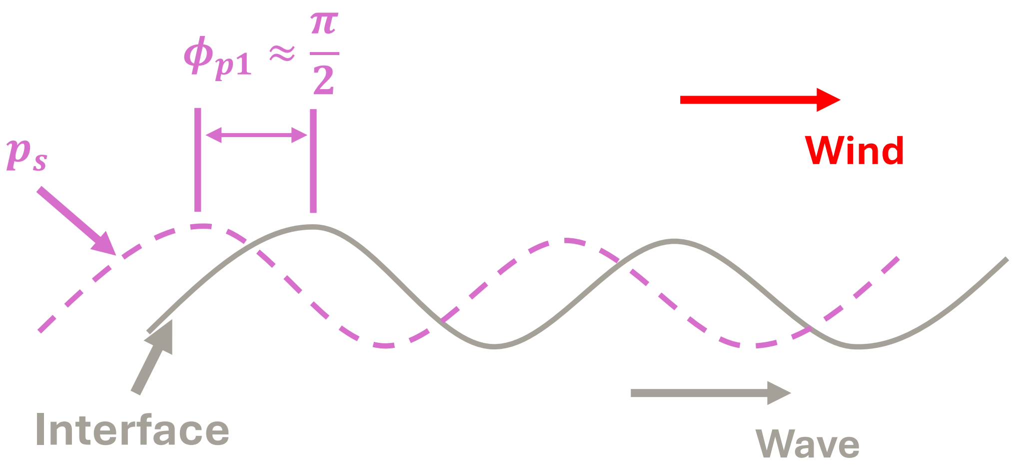
  - Minimum when $|U(\lambda)|\cos(\theta_w) \approx c$
  - Asymmetric about $|U(\lambda)|\cos(\theta_w) \approx c$

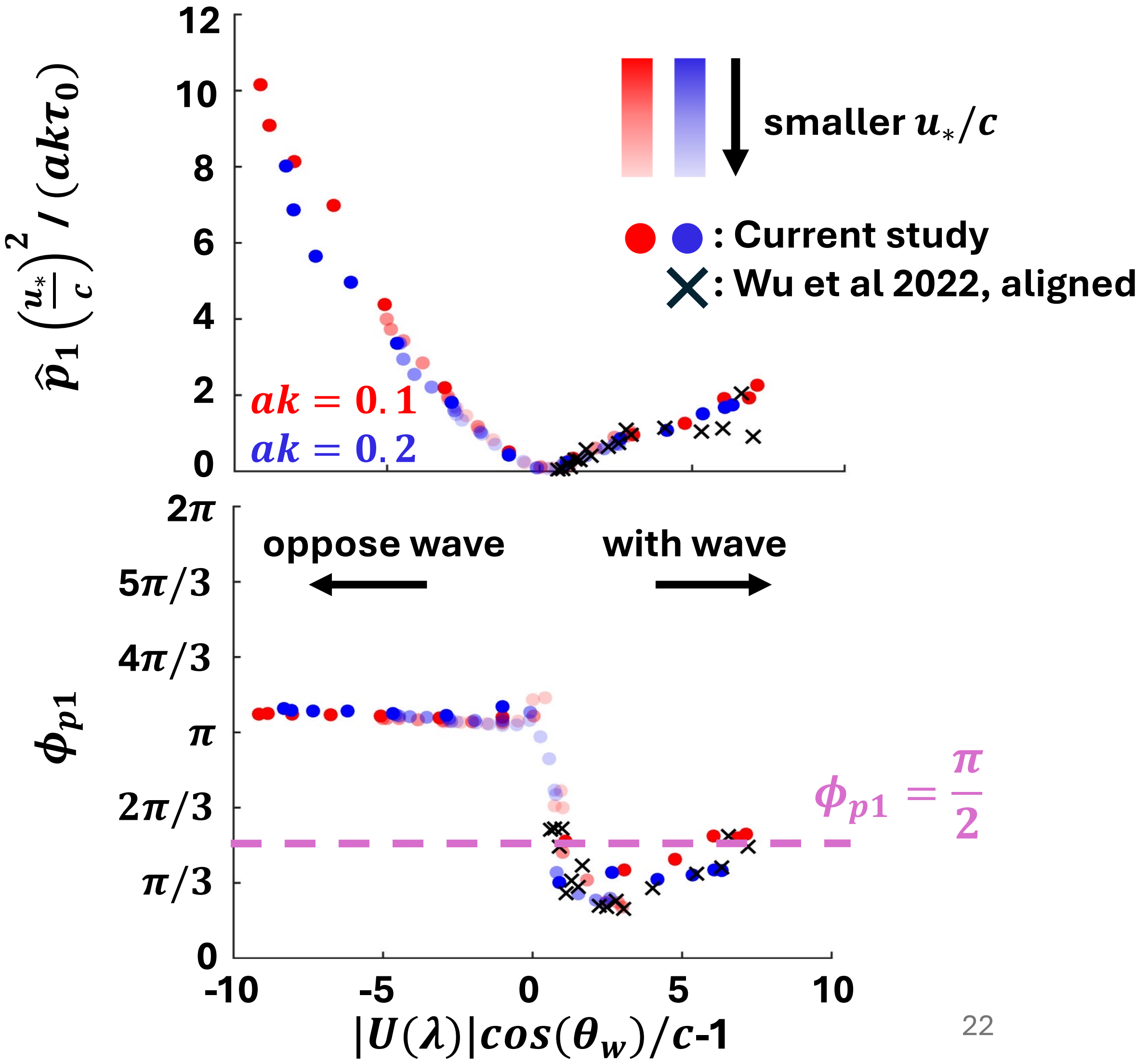


- F_p: Form drag

ϕ_{p1}: Phase lag

- $\sim \frac{\pi}{2}$ for slow wave

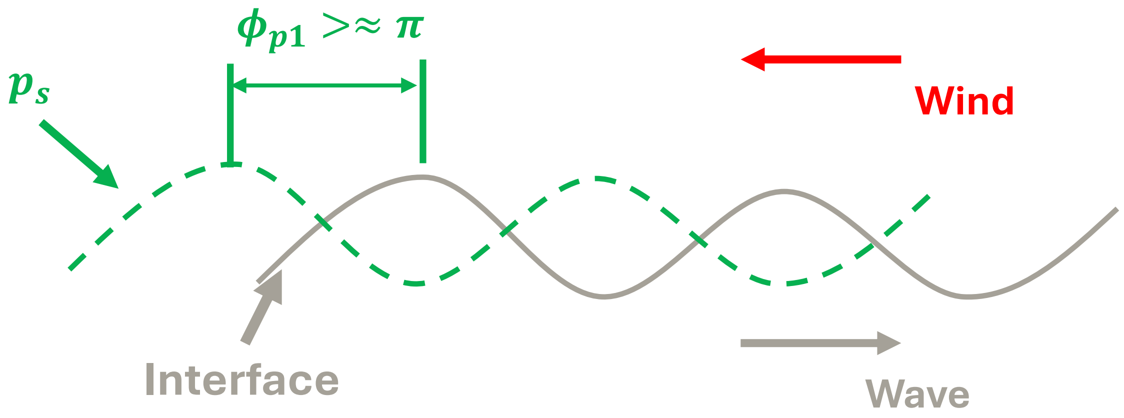


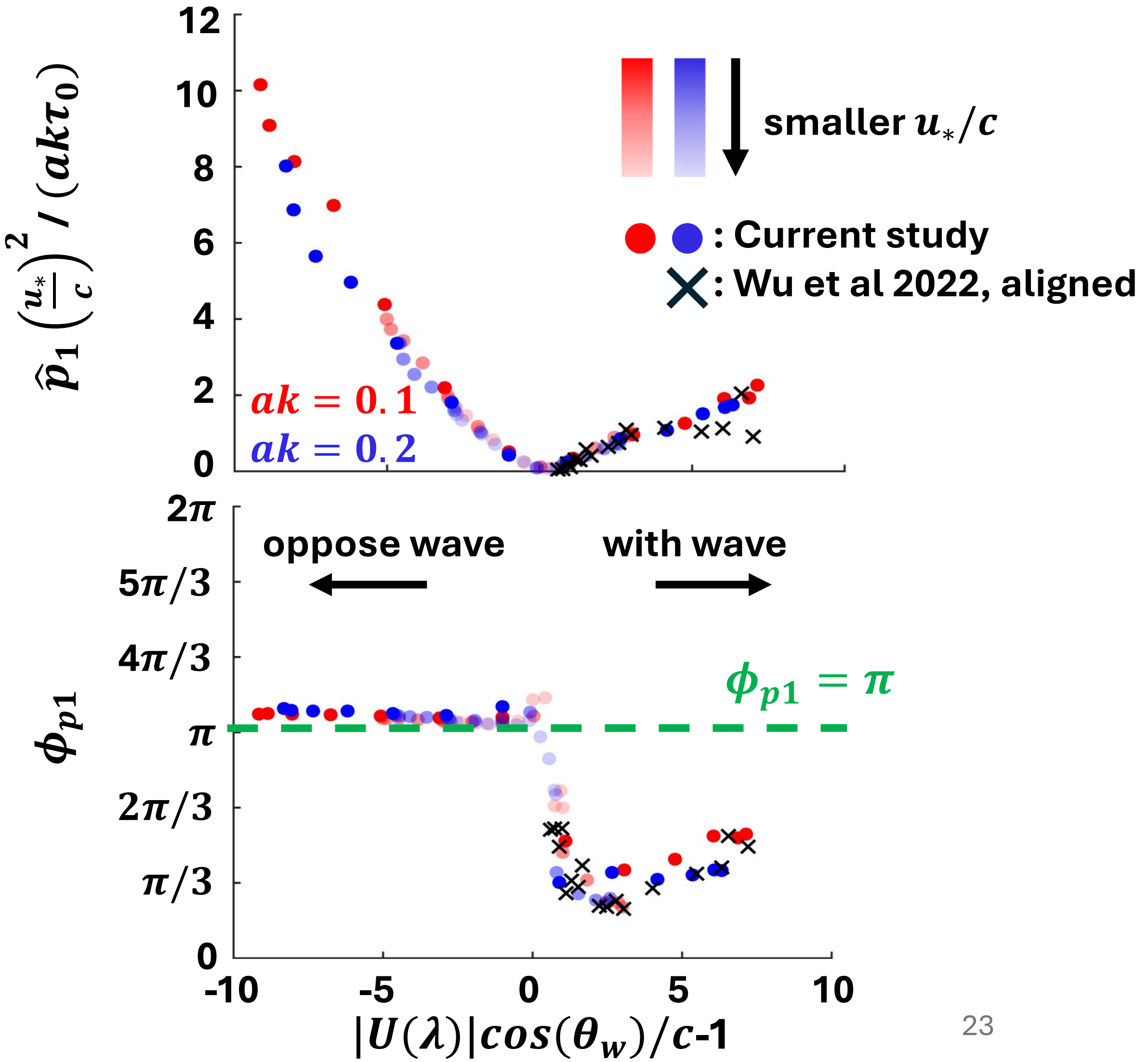


- F_p: Form drag

  ϕ_{p1}: Phase lag

  - $\sim \frac{\pi}{2}$ for slow wave
  - decrease and then increase for faster wave
  - $\sim \pi$ for wind against wave.

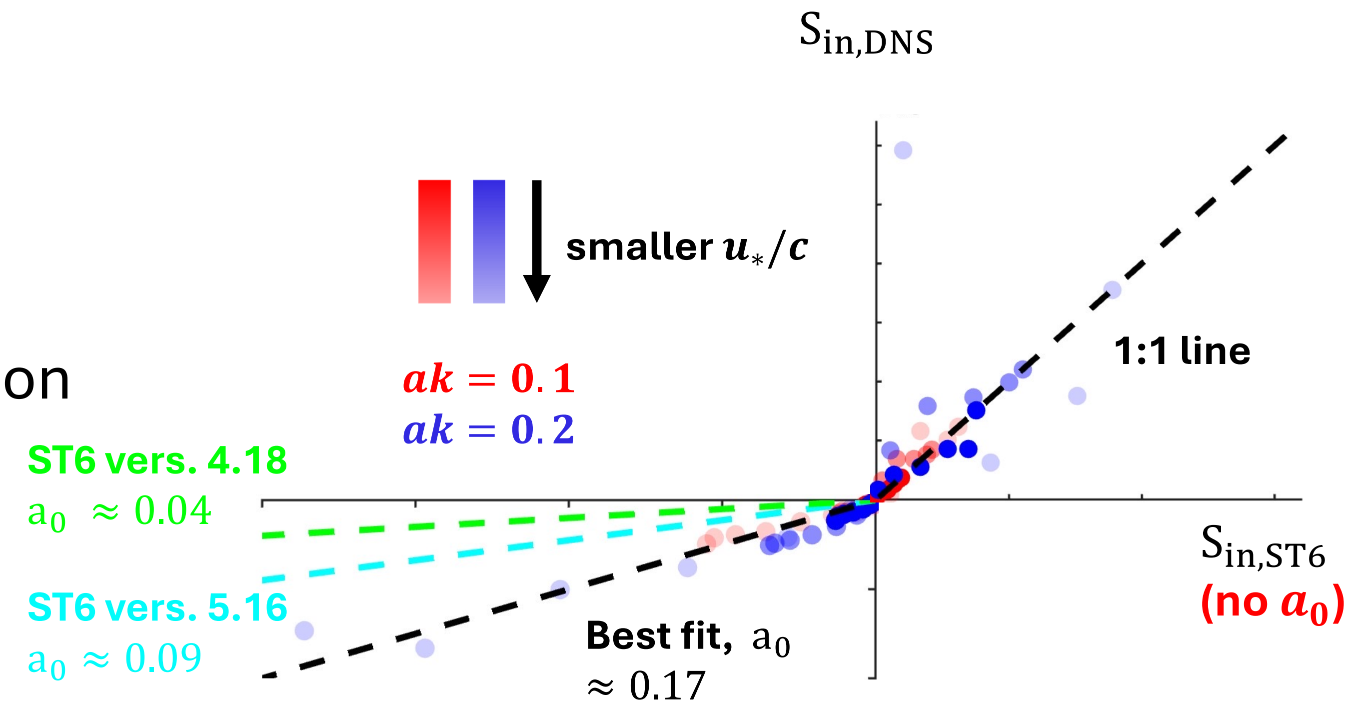


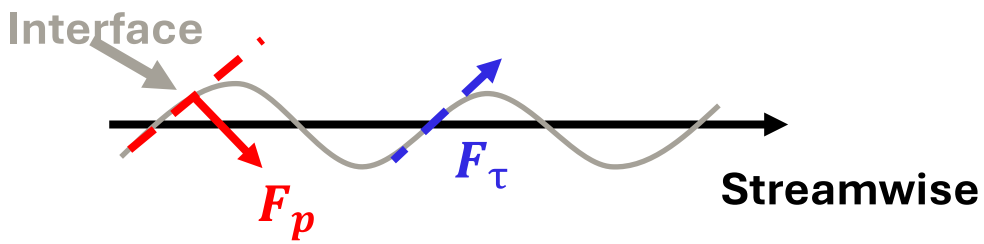


- S_{in}: Pressure input
  - $S_{in} \approx F_p c$
  - WaveWatch III ST6 parametrization

$$S_{in,ST6}(\theta_w) = \begin{cases} \dfrac{\left(\frac{|\boldsymbol{U}(\boldsymbol{\lambda})|}{c}\cos(\theta_w) - 1\right)^2}{\left(\frac{|\boldsymbol{U}(\boldsymbol{\lambda})|}{c} - 1\right)^2} S_{in,ST6}(0) & \text{If positive} \\ a_0 \dfrac{\left(\frac{|\boldsymbol{U}(\boldsymbol{\lambda})|}{c}\cos(\theta_w) - 1\right)^2}{\underbrace{\left(\frac{|\boldsymbol{U}(\boldsymbol{\lambda})|}{c}\right.}_{\text{Wave sheltering}}\left. - 1\right)^2} S_{in,ST6}(0) & \text{If negative} \end{cases}$$

- S_{in}: Pressure input
  - $S_{in} \approx F_p c$
  - WaveWatch III ST6 parametrization
    - Agrees well when wind following
    - Larger a_0 when wind opposing



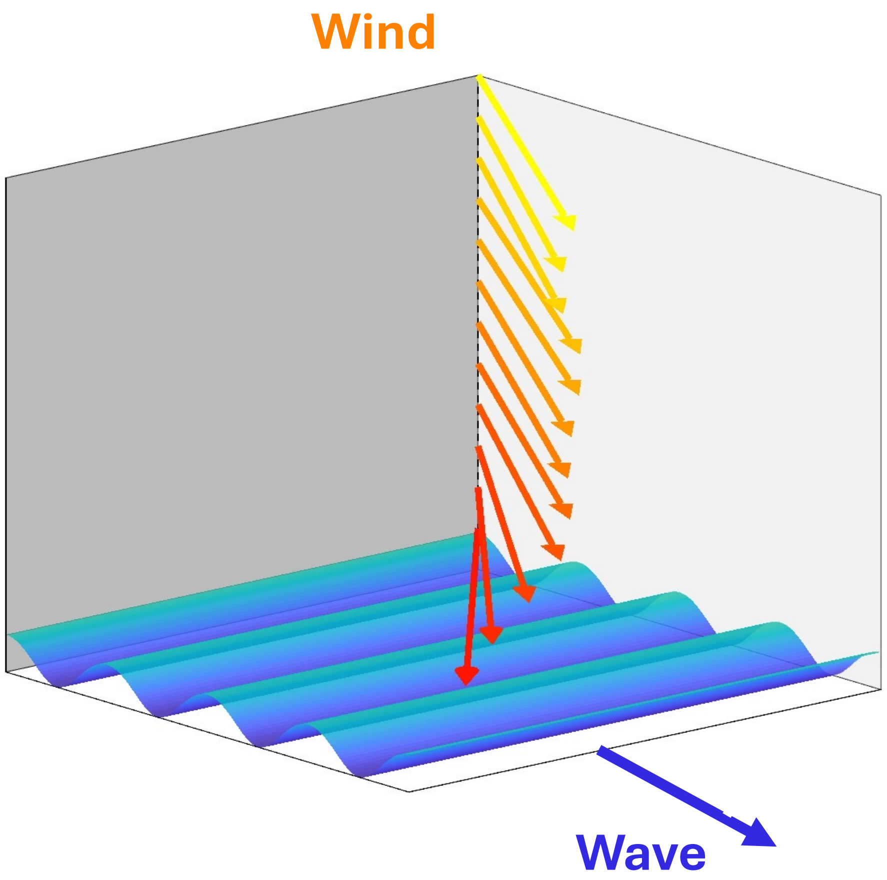


- Surface shear
  - A surface B.C. for ocean models that drives ocean currents
  - Surface shear direction
    - Assumes to be the direction of U_{10}

- Surface shear

However

- Mean wind direction deviates near wave surface.



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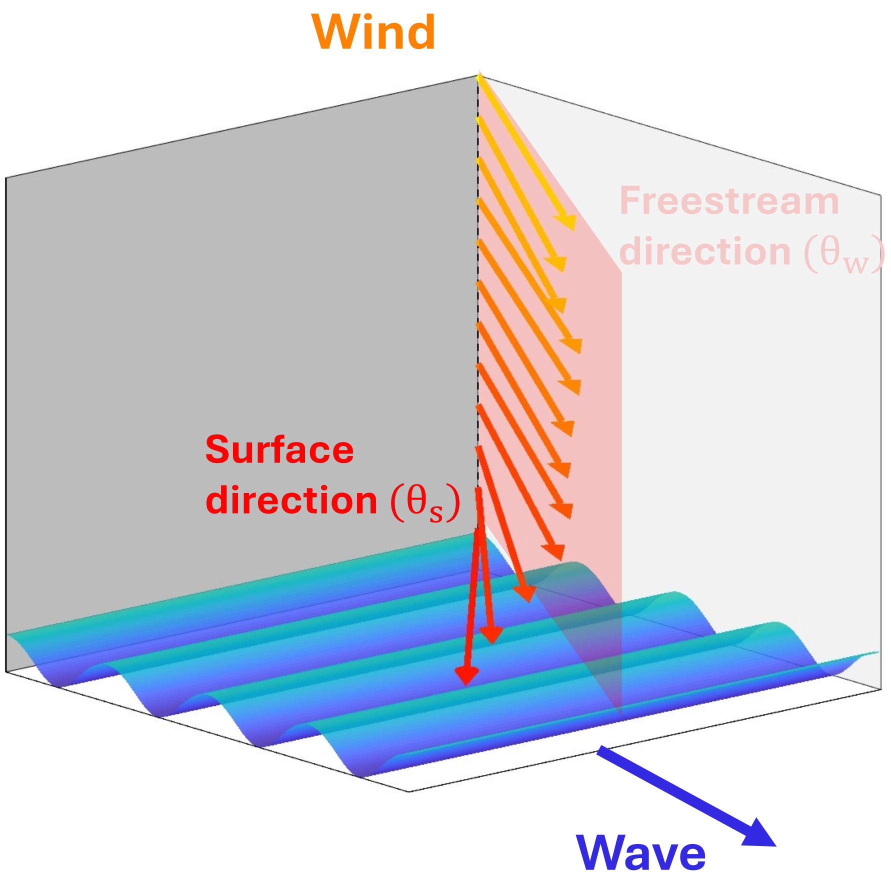

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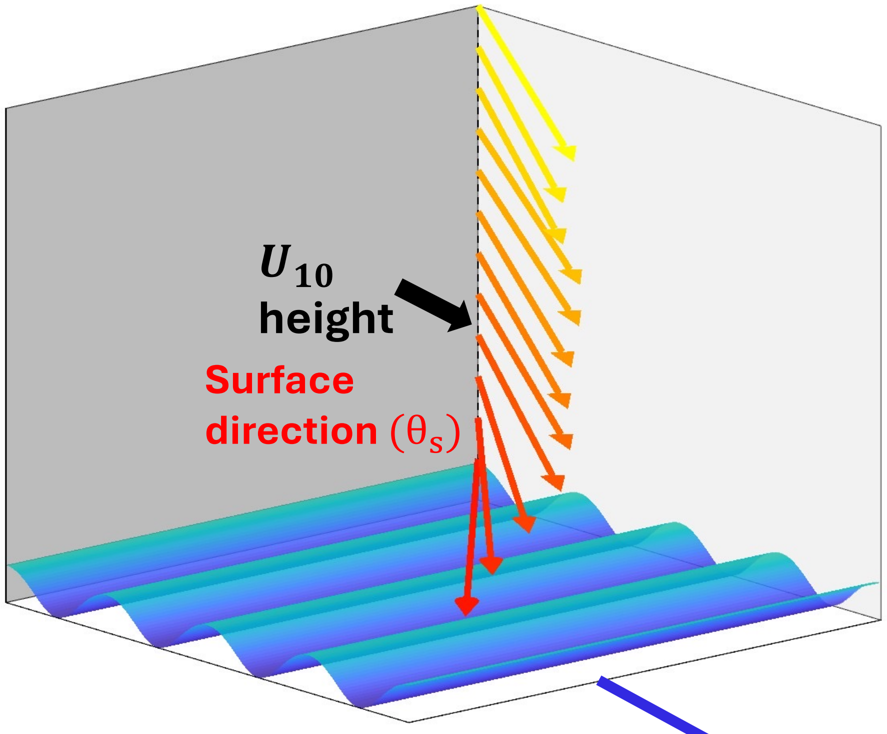

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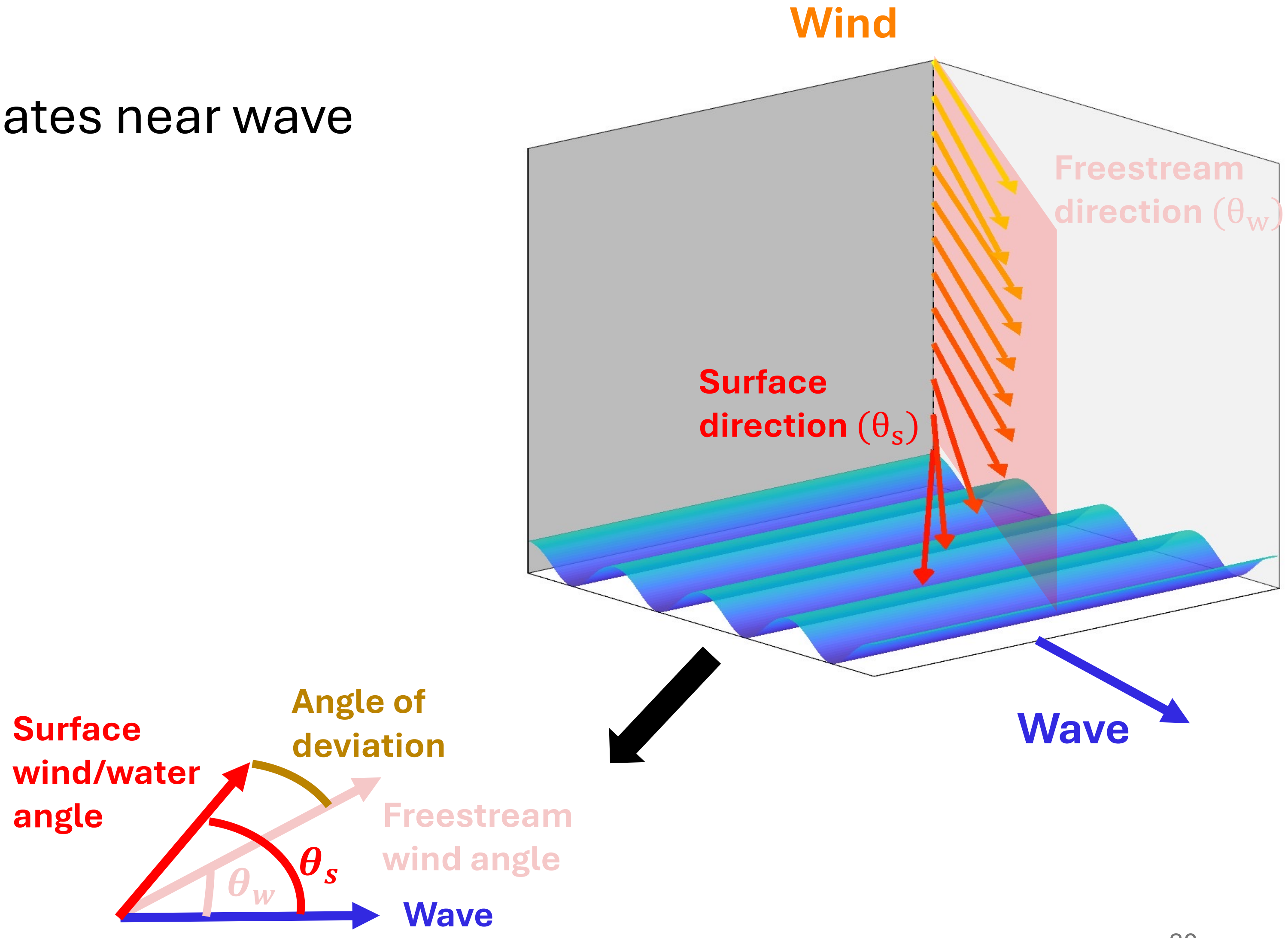
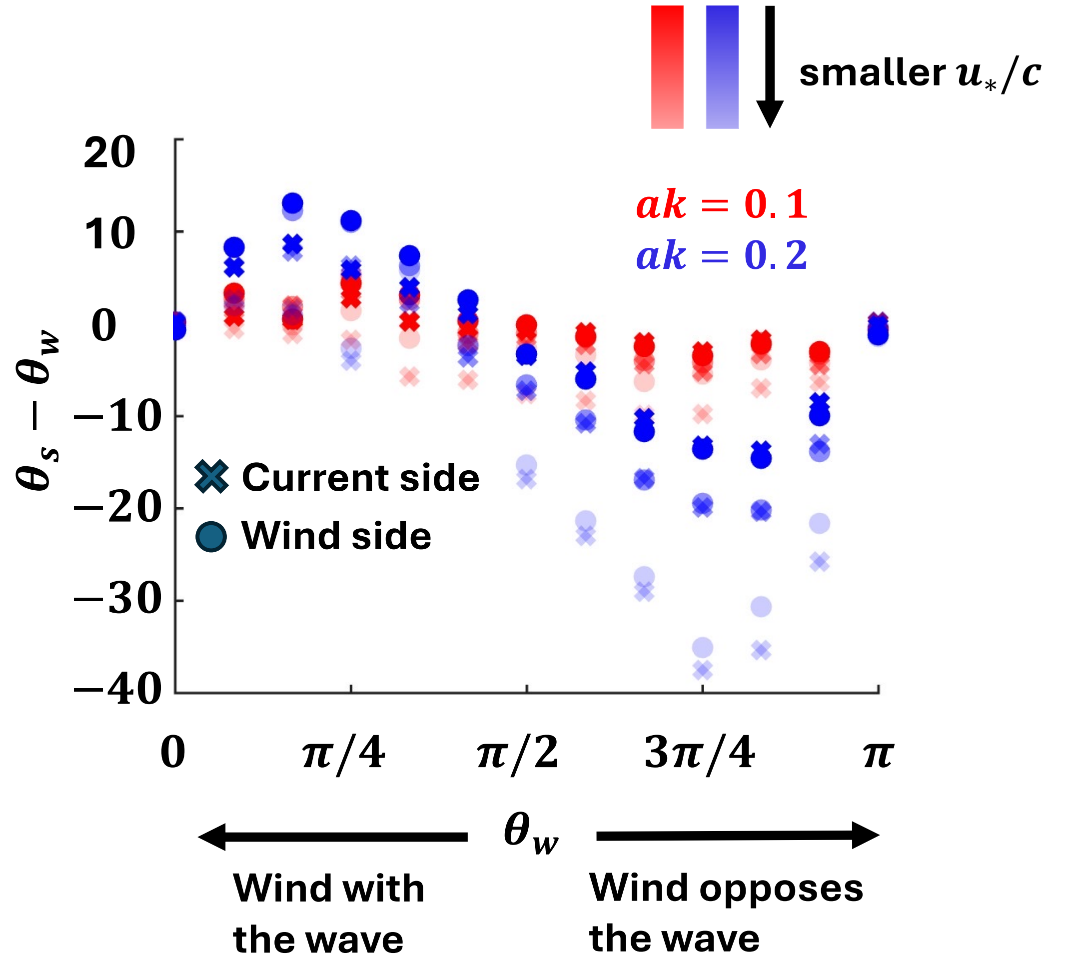
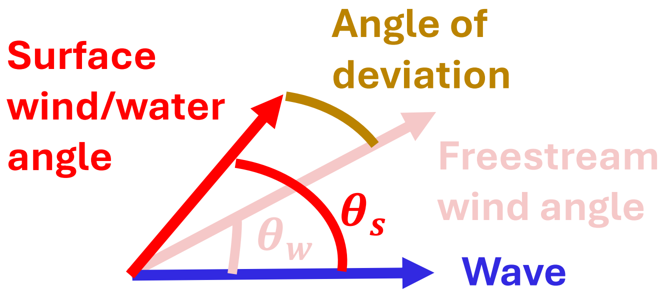


- Surface shear

However

- Mean wind direction deviates near wave surface.
- Surface waterside shear responds to the windside deviation
- Towards spanwise, increase with ak, largest when $\theta_w \approx 45^o$ or 135^o.





- Summary
  - F_p: Form drag
    - Asymmetric
    - $\propto (ak)^2$
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- $\hat{p}_1$: Pressure amplitude
  - minimum at but asymmetric when streamwise wind velocity $\approx c$

- ϕ_{p1}: Phase lag
  - changes with wind-wave regime.

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$\downarrow \times \boldsymbol{c}$

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$\Downarrow \times \boldsymbol{c}$

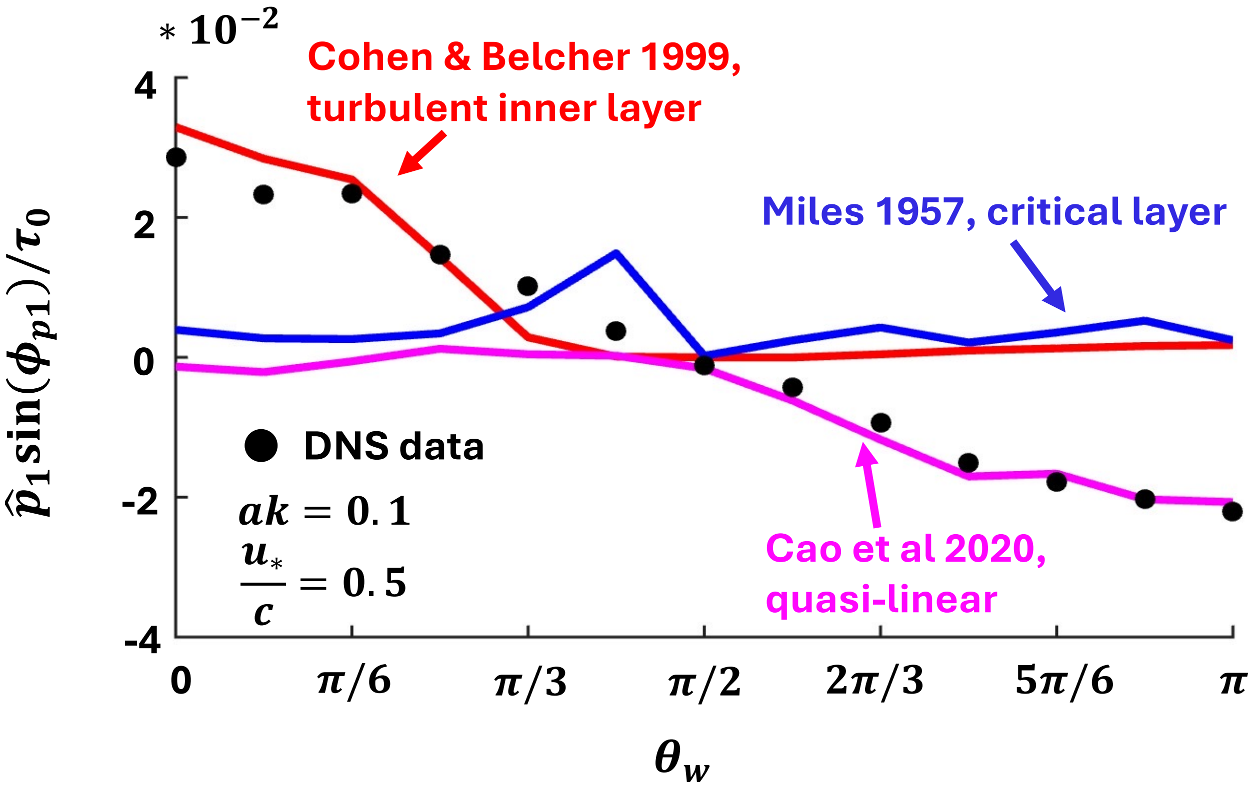
  - S_{in}: Pressure input
    - Agrees well with ST6 when wind following
    - Larger a_0 when wind opposing
  - Surface shear direction
    - Deviates from freestream wind (or U_{10}).
    - Towards spanwise, increase with ak, largest when $\theta_{\mathrm{w}} \approx 45^{\circ}$ or 135°.

- $\hat{p}_1$: Pressure amplitude
  - minimum at but asymmetric when streamwise wind velocity $\approx c$
- ϕ_{p1}: Phase lag
  - changes with wind-wave regime.

- Future study
  - Derive the physical framework for misaligned wind-wave interaction
  - Apply the result to parametrize mean wind profile modification & wave growth / decay prediction.

The following slides are back-up slides

- F_p parametrization
  - $\hat{p}_1 \sin(\phi_{p1})$: Pressure in-phase with wave slope



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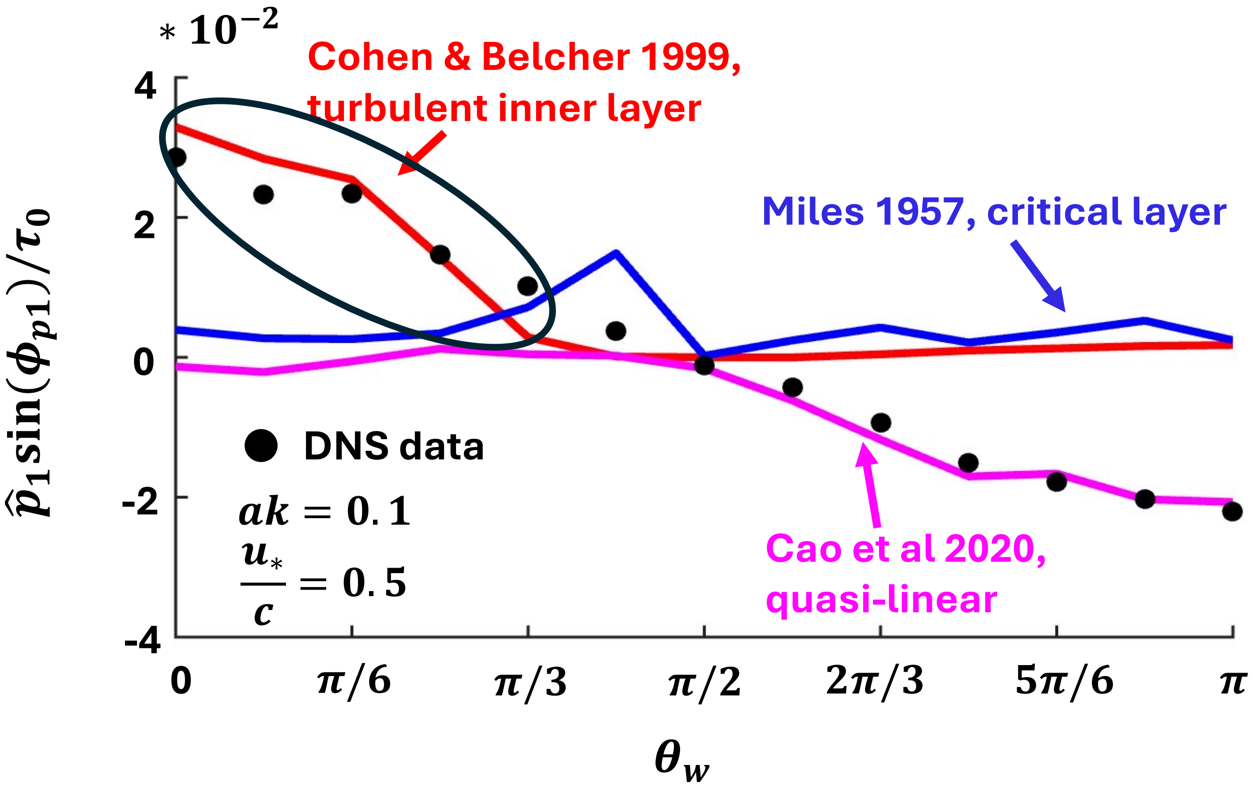
    - Slow wave: turbulence sheltering dominant



- F_p parametrization
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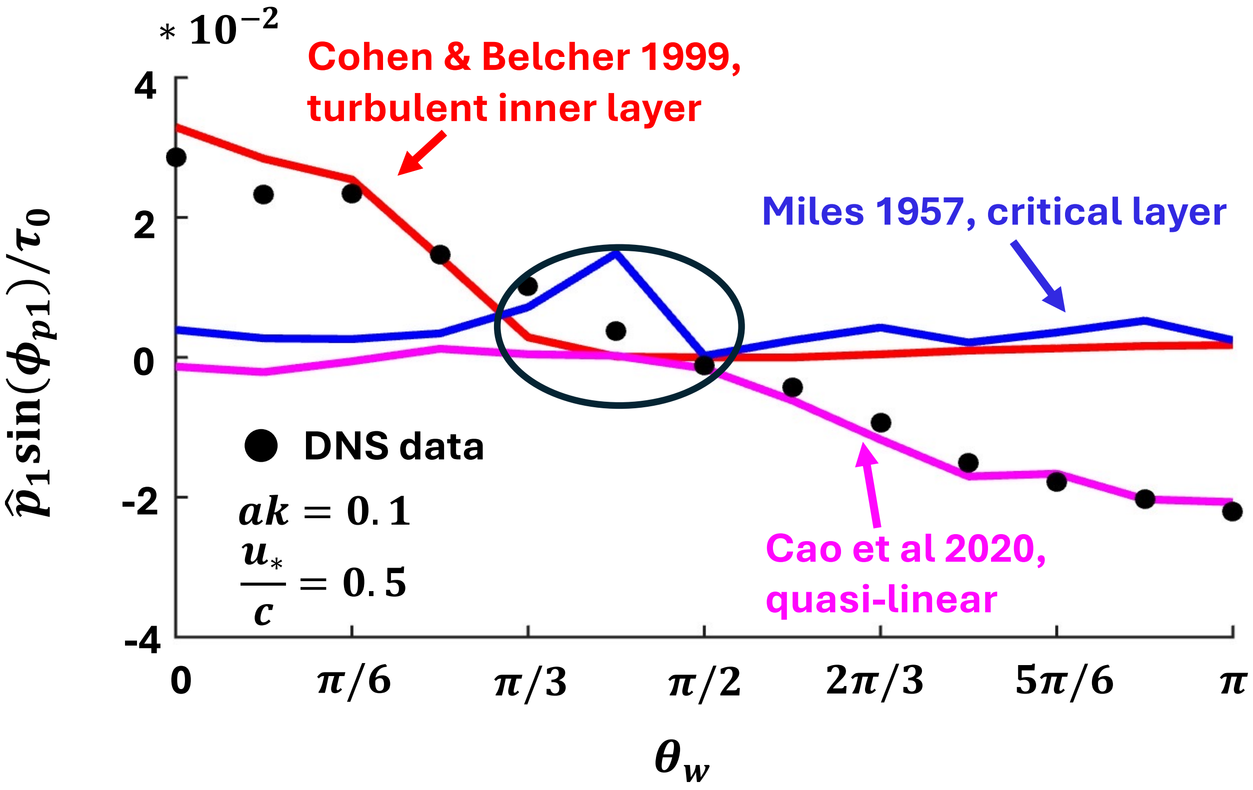
    - Intermediate wave: between turbulence sheltering and critical layer (Kiara et al 2007)



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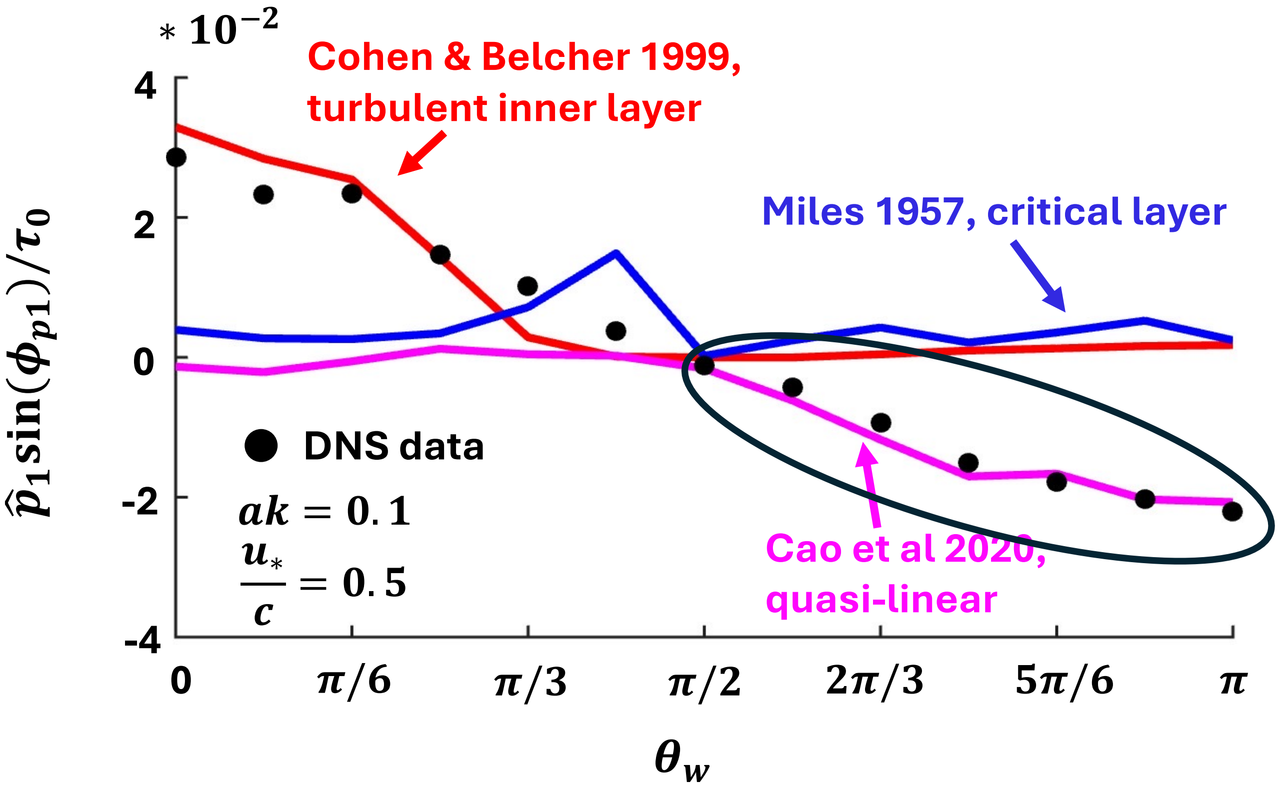
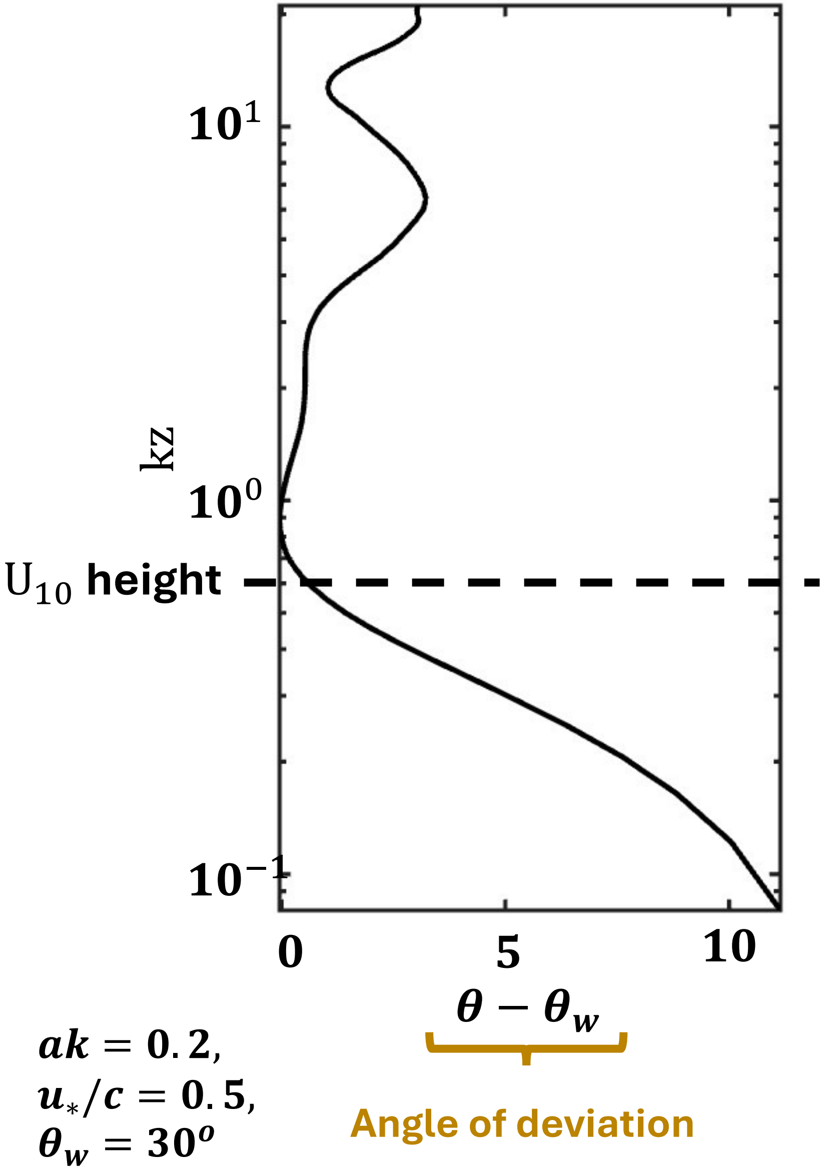
    - Opposing wave: quasi-linear process

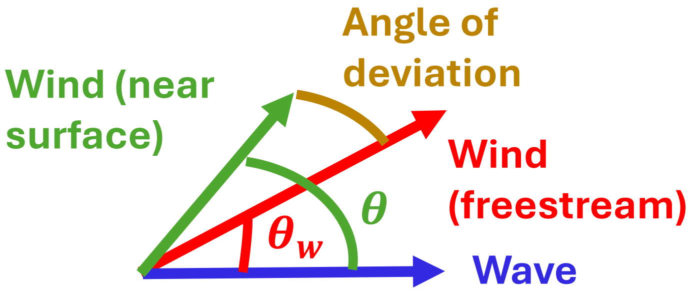


- Surface shear

However

- Mean wind direction deviates near wave surface (Husain et al 2022)

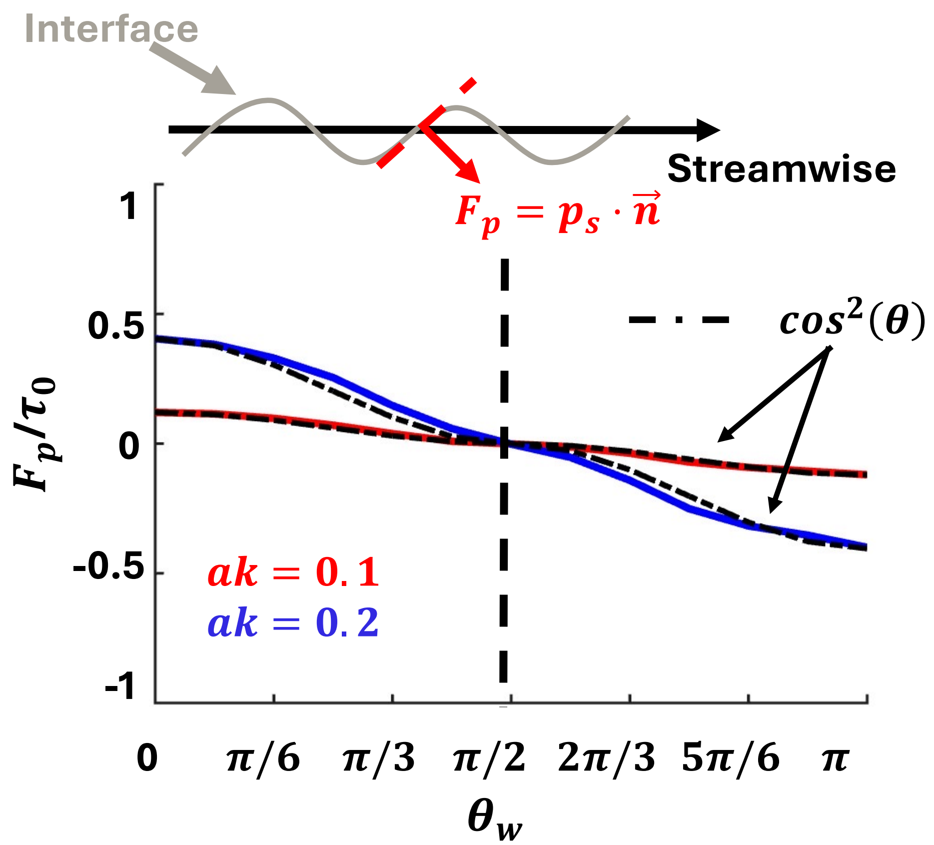


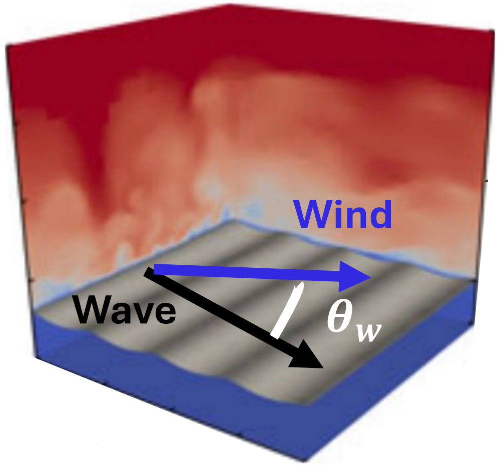


- Precursor runs

  F_p: **Form drag**

  - Symmetric
  - Increase with increase ak
  - Follows $\cos^2(\theta_w)$ for small slopes





- Precursor runs

F_p:

  - Symmetric
  - Increase with increase ak
  - Follows $\cos^2(\theta_w)$ for small slopes

F_τ:

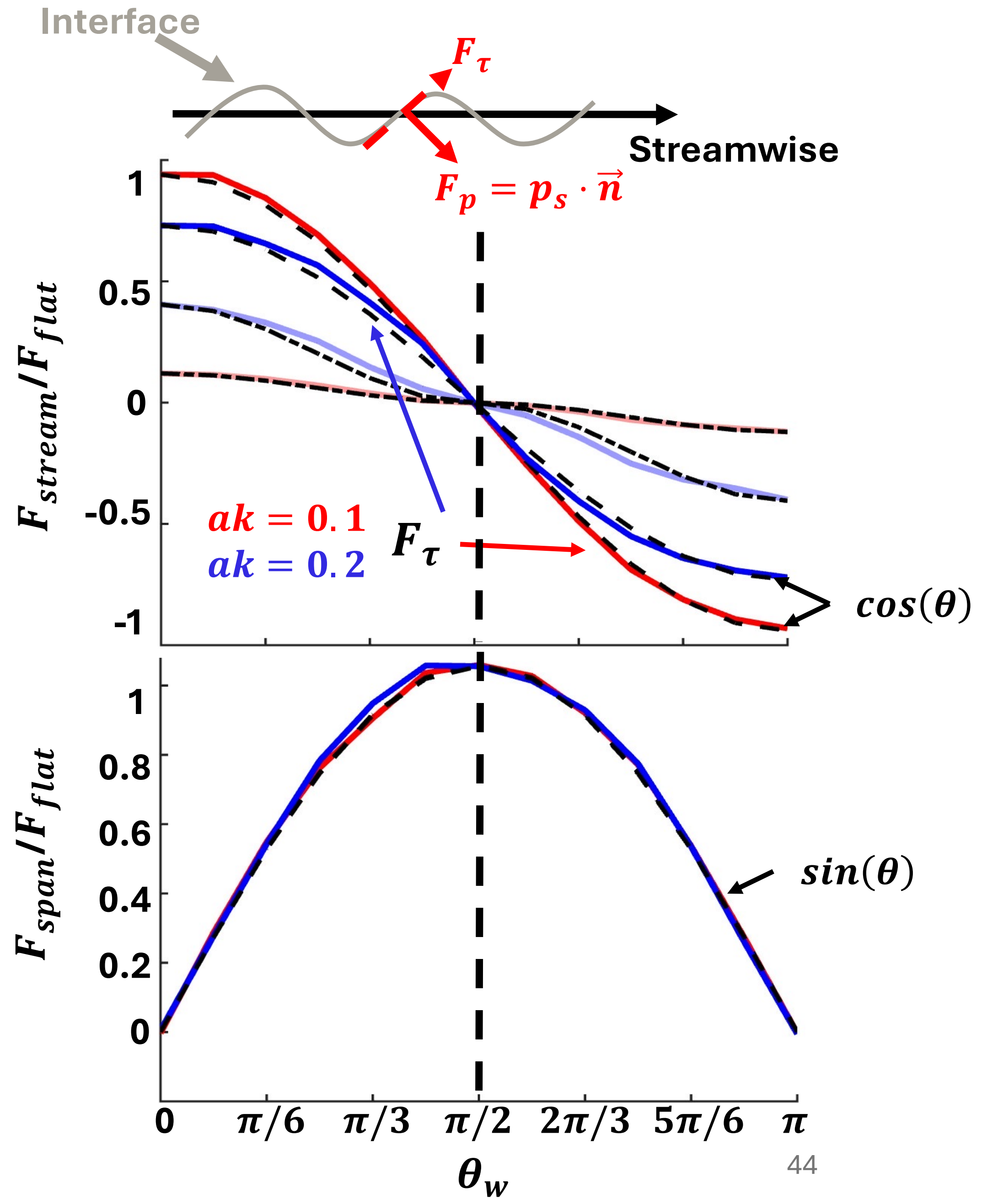
  - Symmetric

  Streamwise:

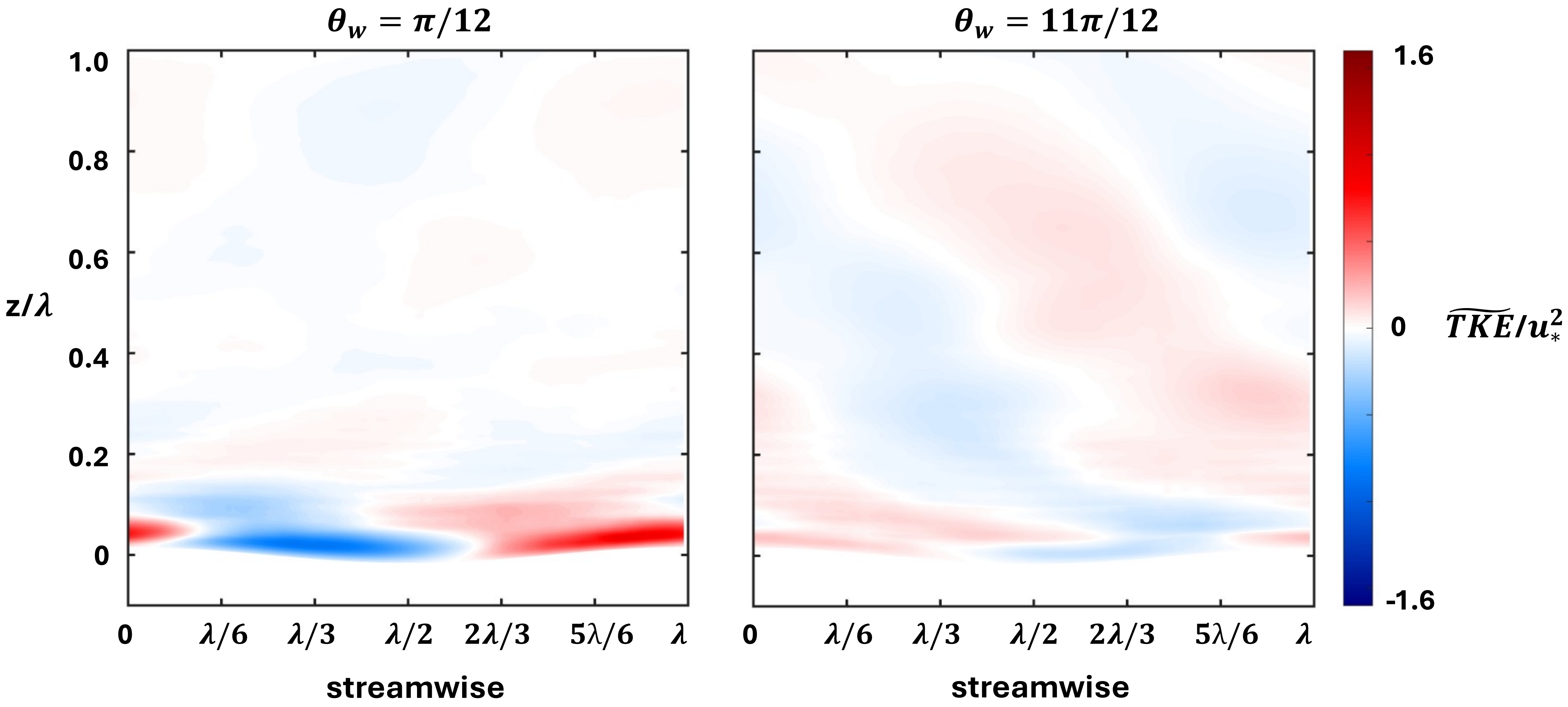
    - Decrease with increase ak
    - Follows $cos(\theta_w)$

  Spanwise:

    - Follows $\sin(\theta_w)$



- TKE profile



$\theta_w = \pi/12$
$\theta_w = 11\pi/12$
1.0
0.8
0.6
z/λ
0.4
0.2
0
0
λ/6
λ/3
λ/2
2λ/3
5λ/6
λ
streamwise
0
λ/6
λ/3
λ/2
2λ/3
5λ/6
λ
streamwise
1.6
0
-1.6
$\widetilde{TKE}/u_*^2$

- Two modeling stages

Precursor → **Fully coupled**

Precursor

- Initialize with wind + surface elevation
- One phase (air)
- Run until quasi-steady

Fully coupled

- Initialize with quasi-steady precursor run output
- Prescribe initial water velocity via wave theory
- Two-phase, fully coupled

- Inner layer height vs critical layer