2026 International Ocean Vector Winds Science Team (IOVWST) Meeting

Kinematics of Gravity–Capillary Waves Above an Evolving Underwater Current

Clara Martin Blanco[1], Nicolo Scapin[1,5], Jiarong Wu[1], Stéphane Popinet[2], Tom Farrar[3], Bertrand Chapron[4] and Luc Deike[1,5]

[1]Department of Mechanical and Aerospace Engineering, Princeton University, Princeton, NJ 08544, USA
[2]Institut Jean Le Rond d'Alembert, CNRS UMR 7190, Sorbonne Université, Paris 75005, France,
[3]Woods Hole Oceanographic Institution, Woods Hole, MA 02543, USA
[4]Ifremer, 29280 Plouzane,´France
[5]High Meadows Environmental Institute, Princeton University, Princeton, NJ 08544, US

PRINCETON UNIVERSITY

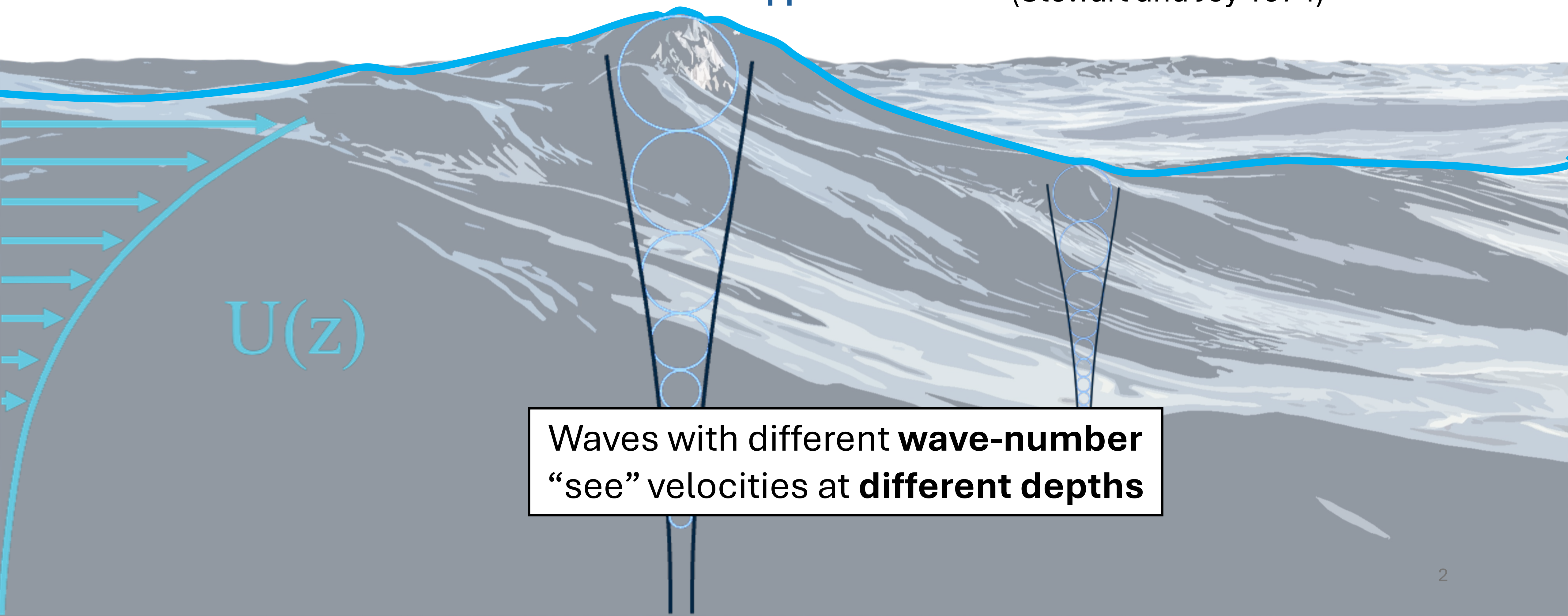
Introduction

$$\omega(k) = \sqrt{gk + \frac{\sigma}{\rho}k^3} + u_{eff}(k)k$$

Doppler shift

With $u_{eff}(k) = 2k \int_0^\infty u_L(z) e^{-2kz} dz$

(Stewart and Joy 1974)



Waves with different **wave-number** “see” velocities at **different depths**

Non-linear waves’ propagation and growth across scales

Part I: Kinematics
of modes k

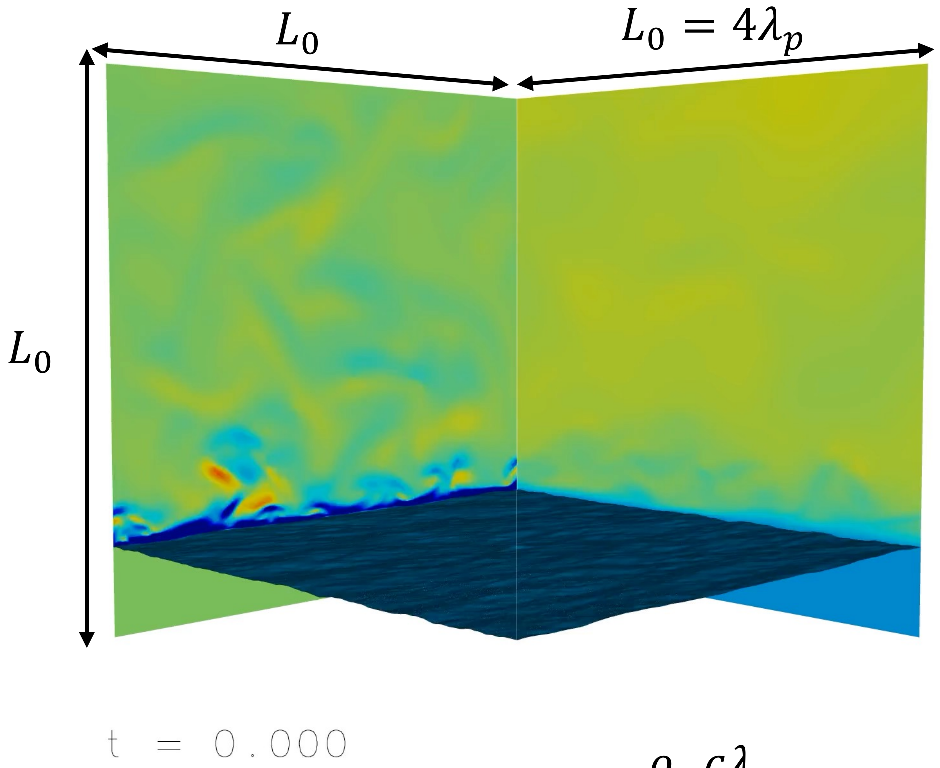
How are non-linear waves Doppler **shifted**?

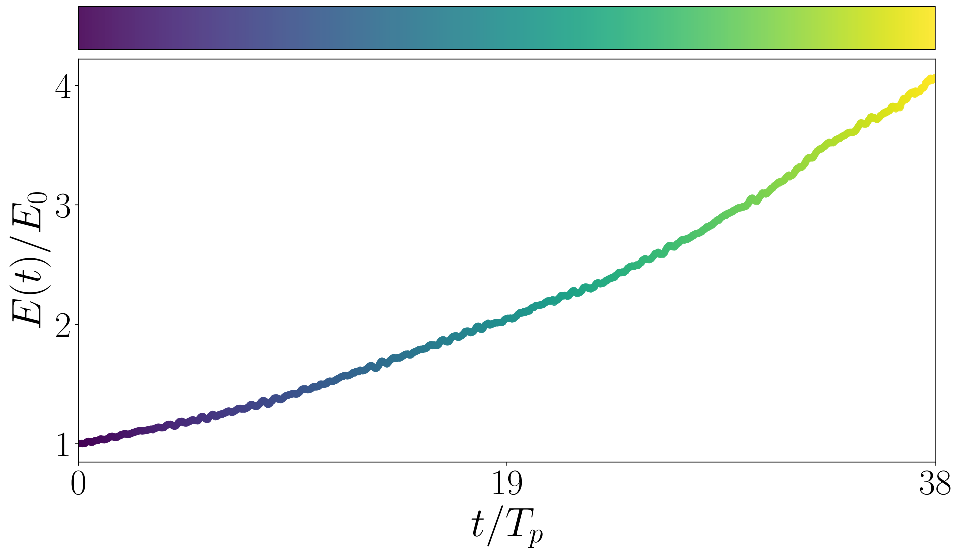
Part II: Dynamics
of modes k

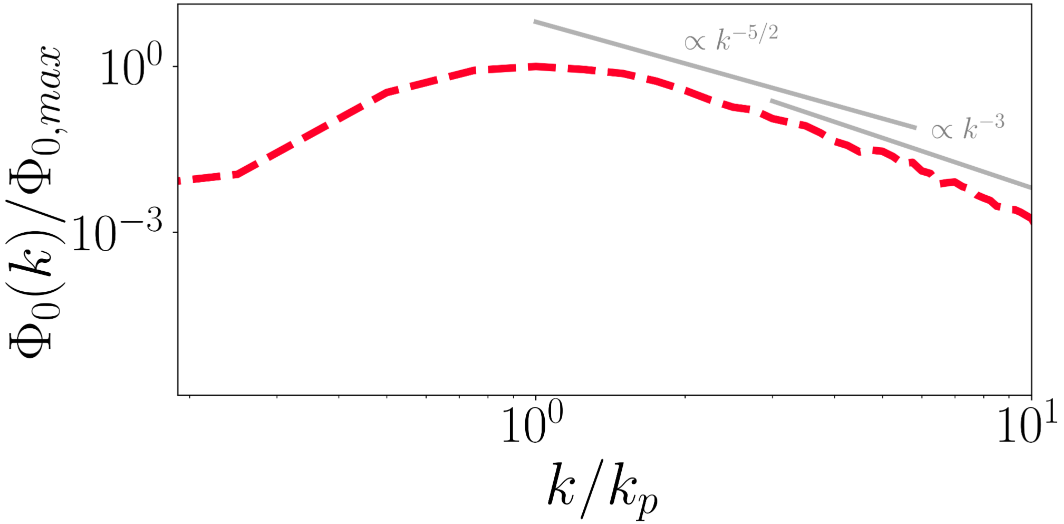
How non-linear Doppler shifted waves **grow**?

DNS Two-Phase Flow setup (Adapted from Wu et al. 2022)

Temporally evolving **broadbanded wavefield, coupled** with a **turbulent boundary layers** on both the **air** and the **water**







$u_*/c = 0.25,\ 0.5$

$k_pH_s = 0.08,\ 0.16$

$$Re_w = \frac{\rho_w c \lambda_p}{\mu_w} = 2.5 \cdot 10^4$$

$$Bo = \frac{(\rho_w - \rho_a)g}{k_p^2 \sigma} = 200$$

Basilisk: Open-source solver (developed by Stephane Popinet): **http://basilisk.fr/**

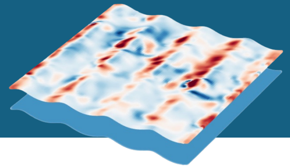
Non-linear waves' propagation and growth across scales

Part I: Kinematics

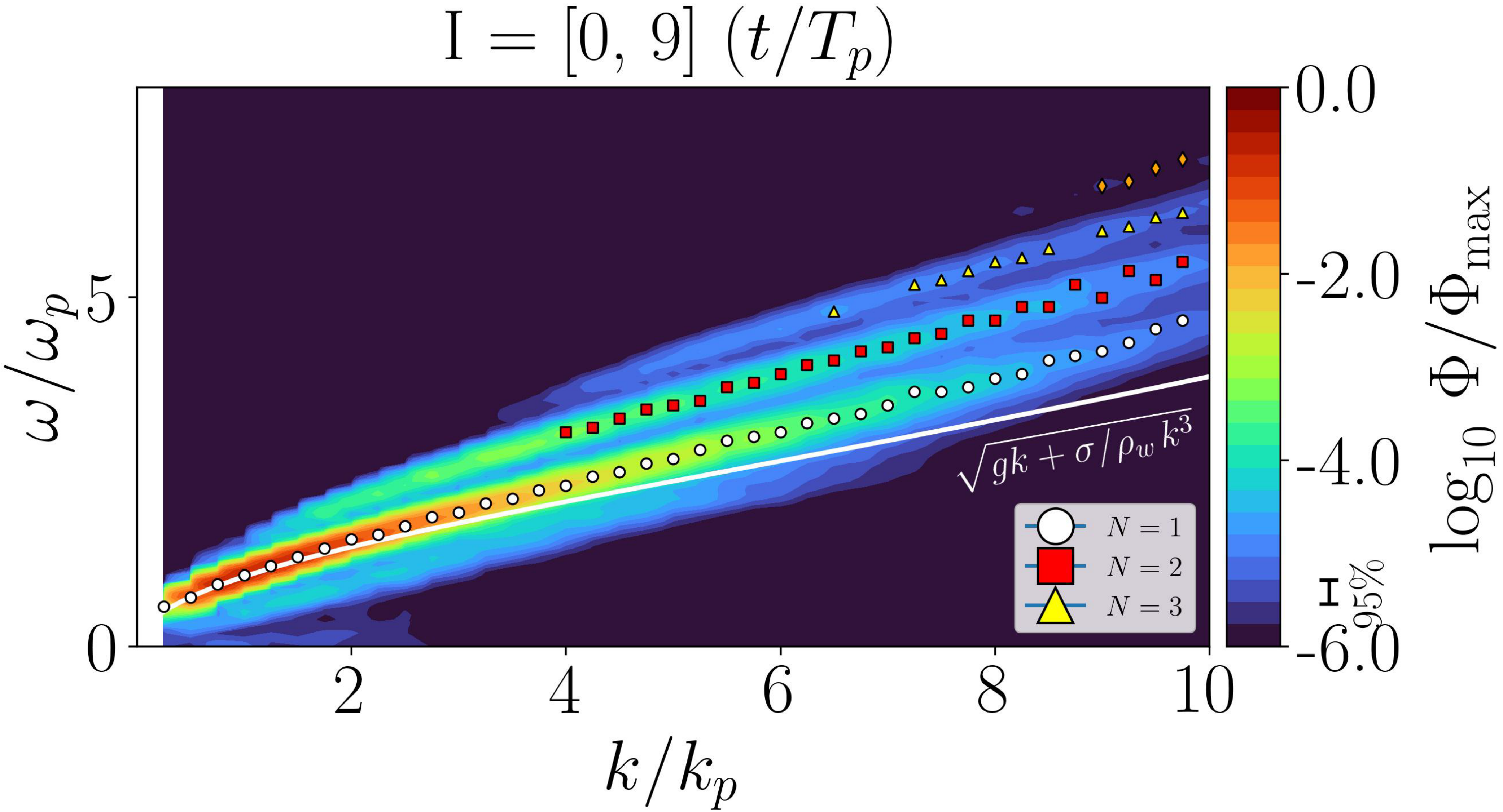
of modes k

How are non-linear waves Doppler **shifted**?

Spatio-Temporal FFT analysis

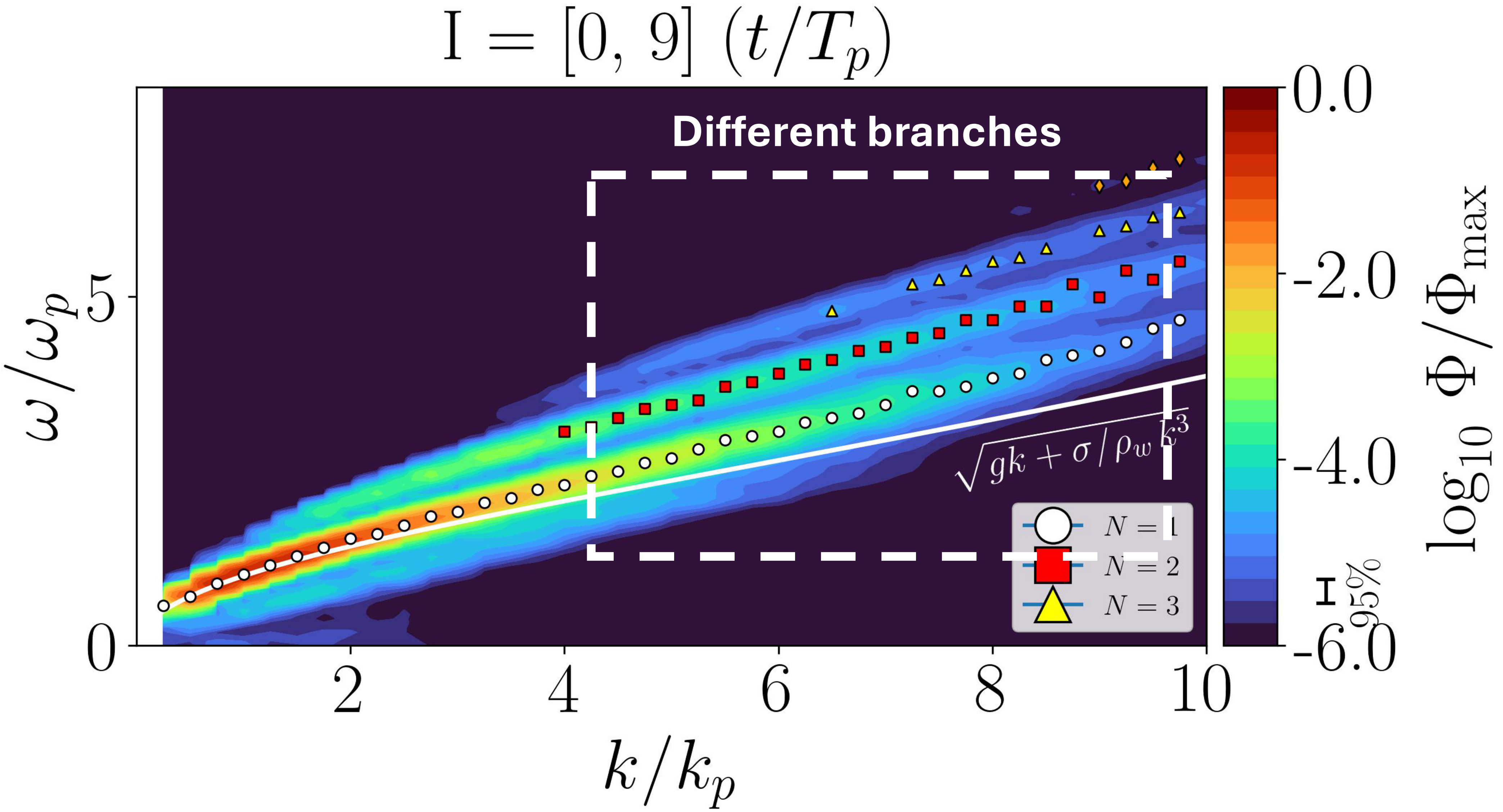


$\eta(x, y, t) \xrightarrow{\text{FFT}} \Phi(k, \omega, t)$



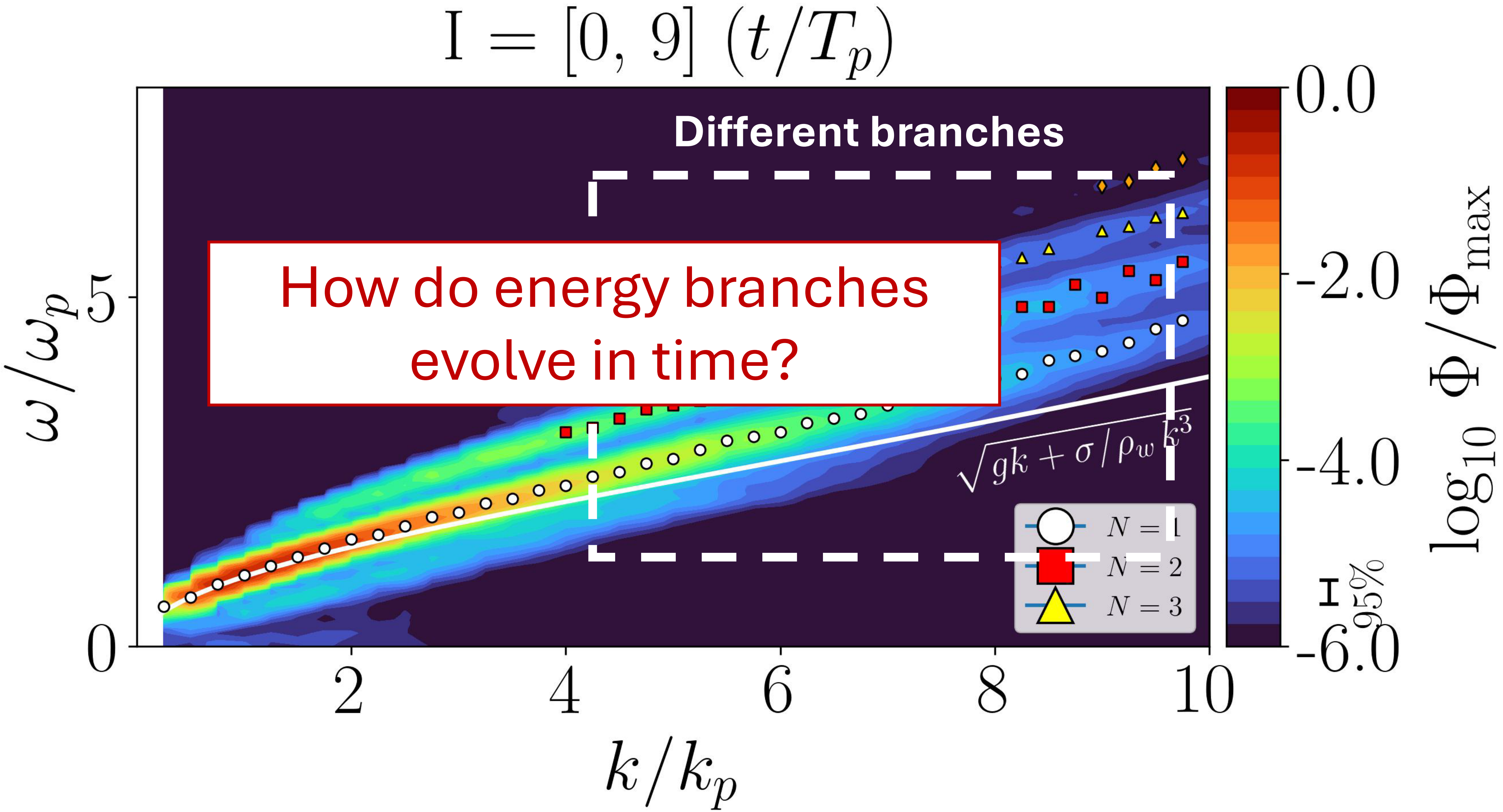
Higher order harmonics

Energy is contained in multiple branches including the linear **dispersion relation**

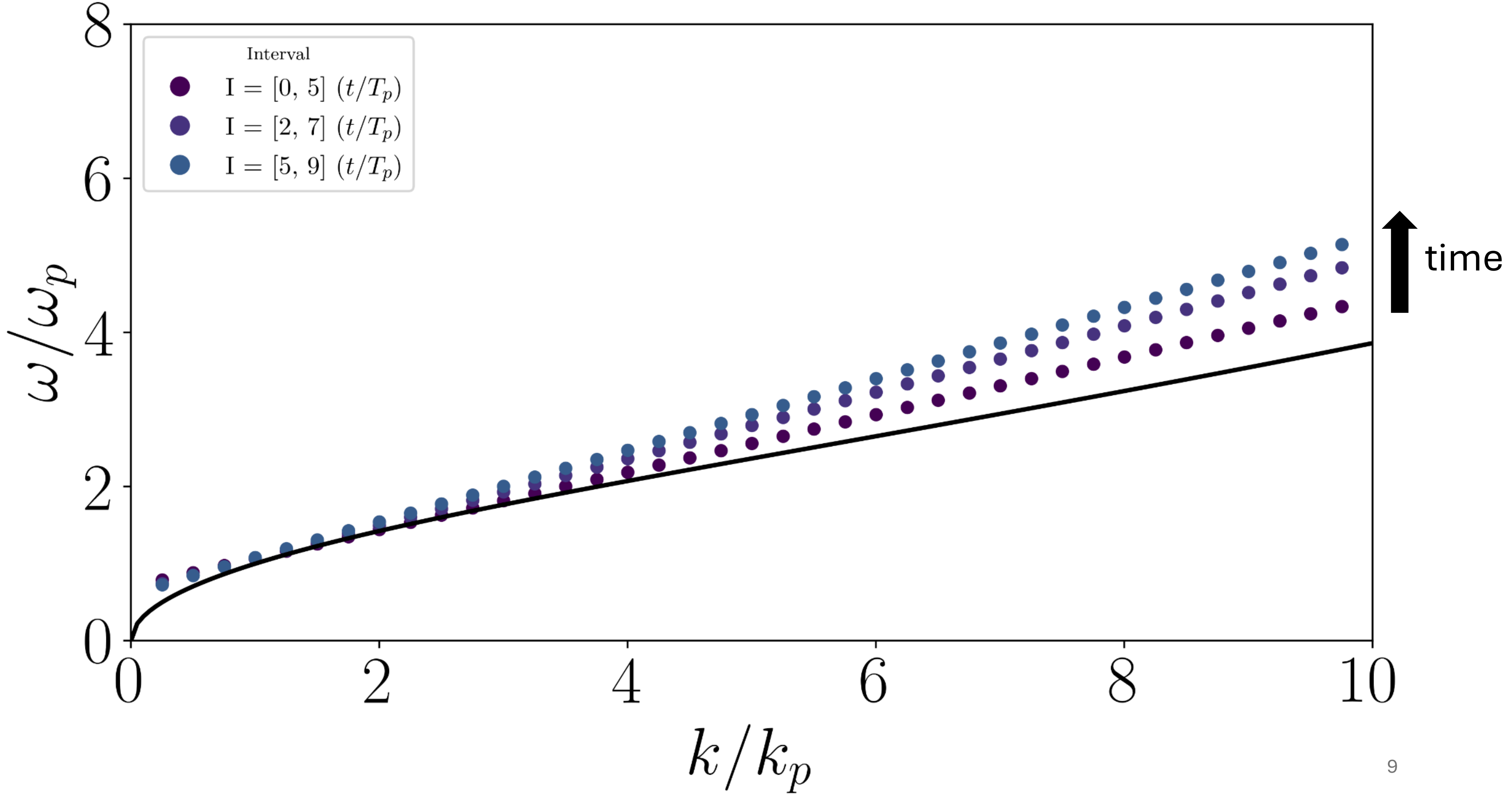


Higher order harmonics

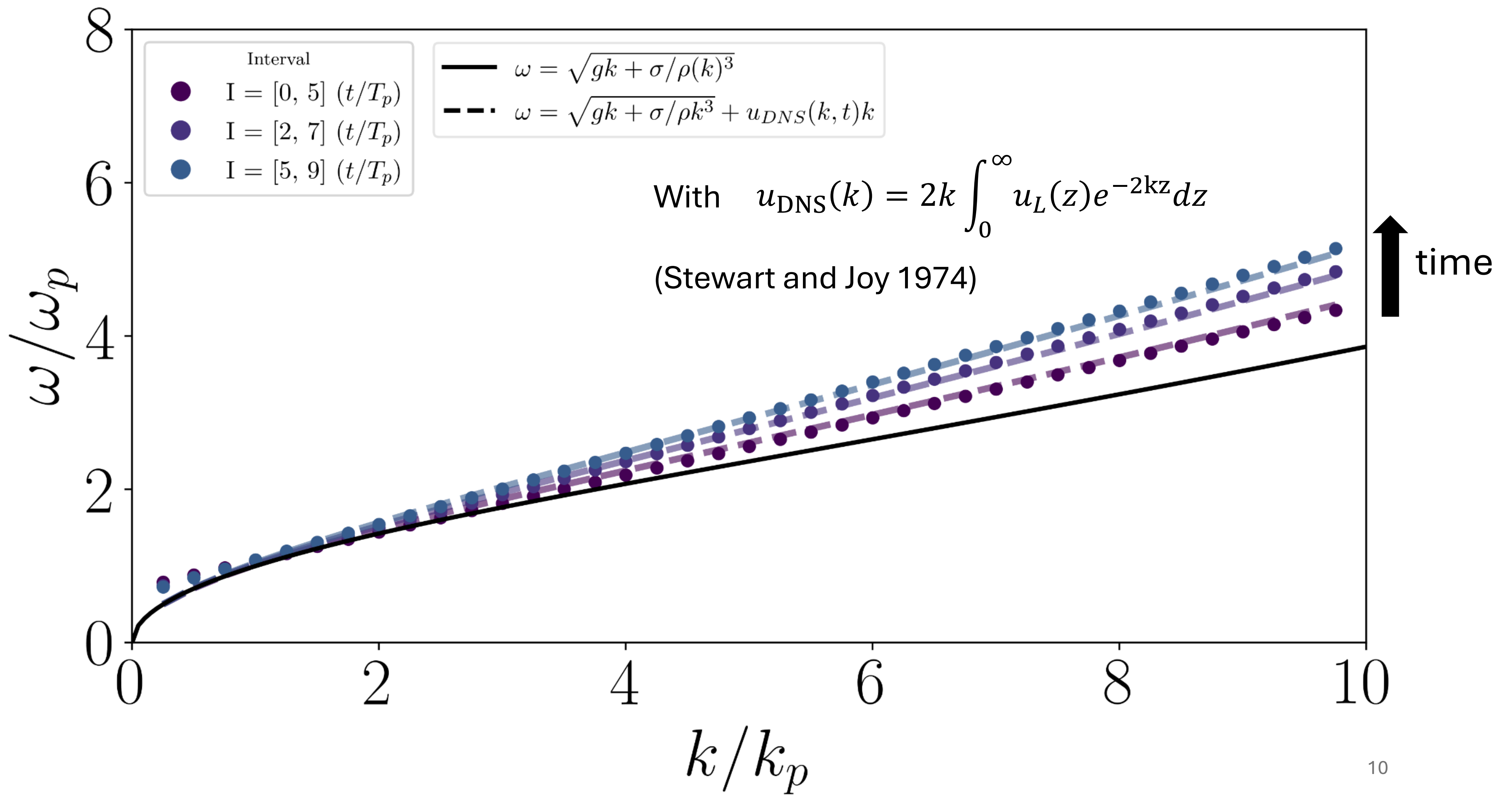
Energy is contained in multiple branches including the linear **dispersion relation**



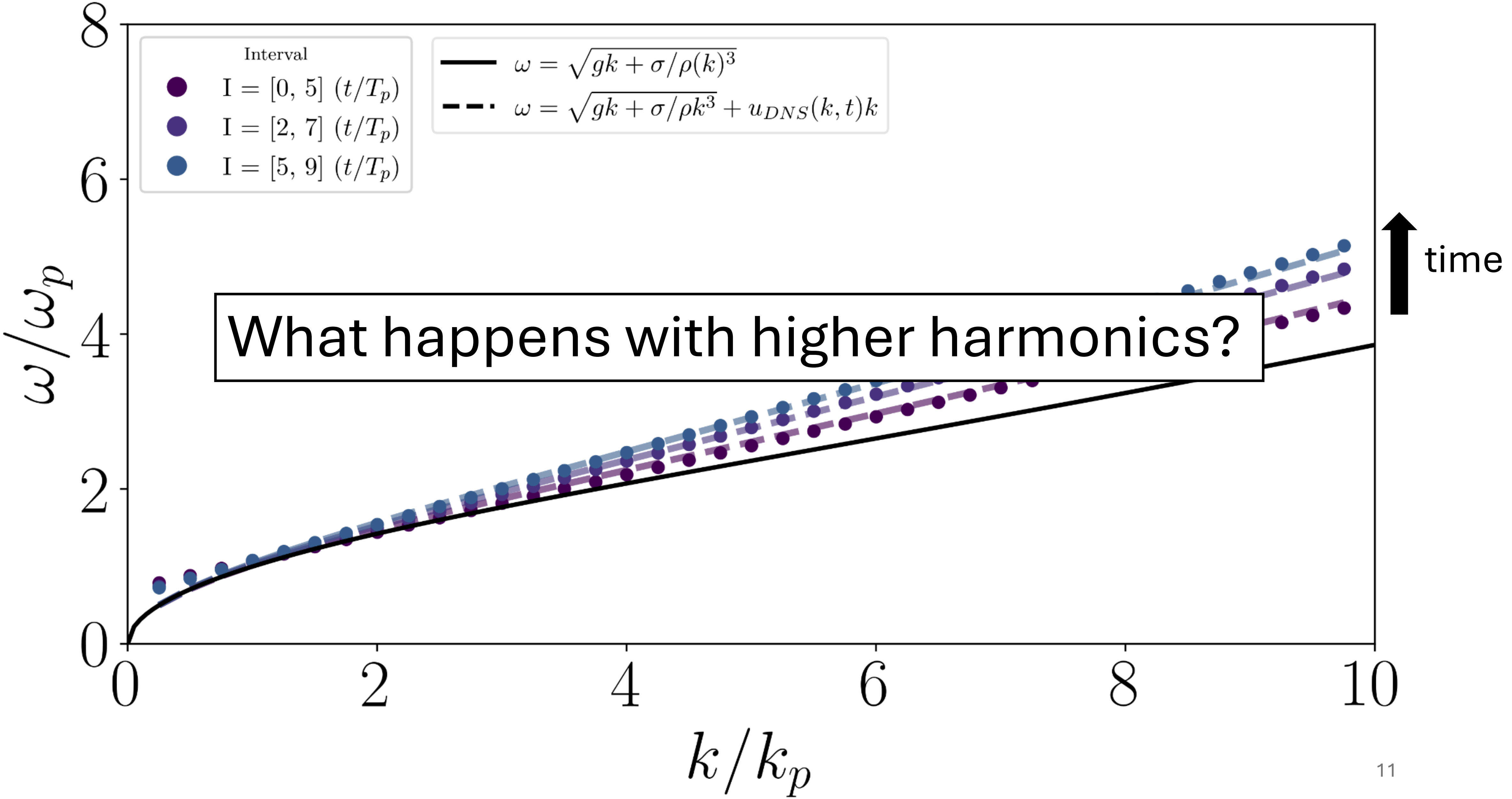
Shift from Linear Dispersion Relationship



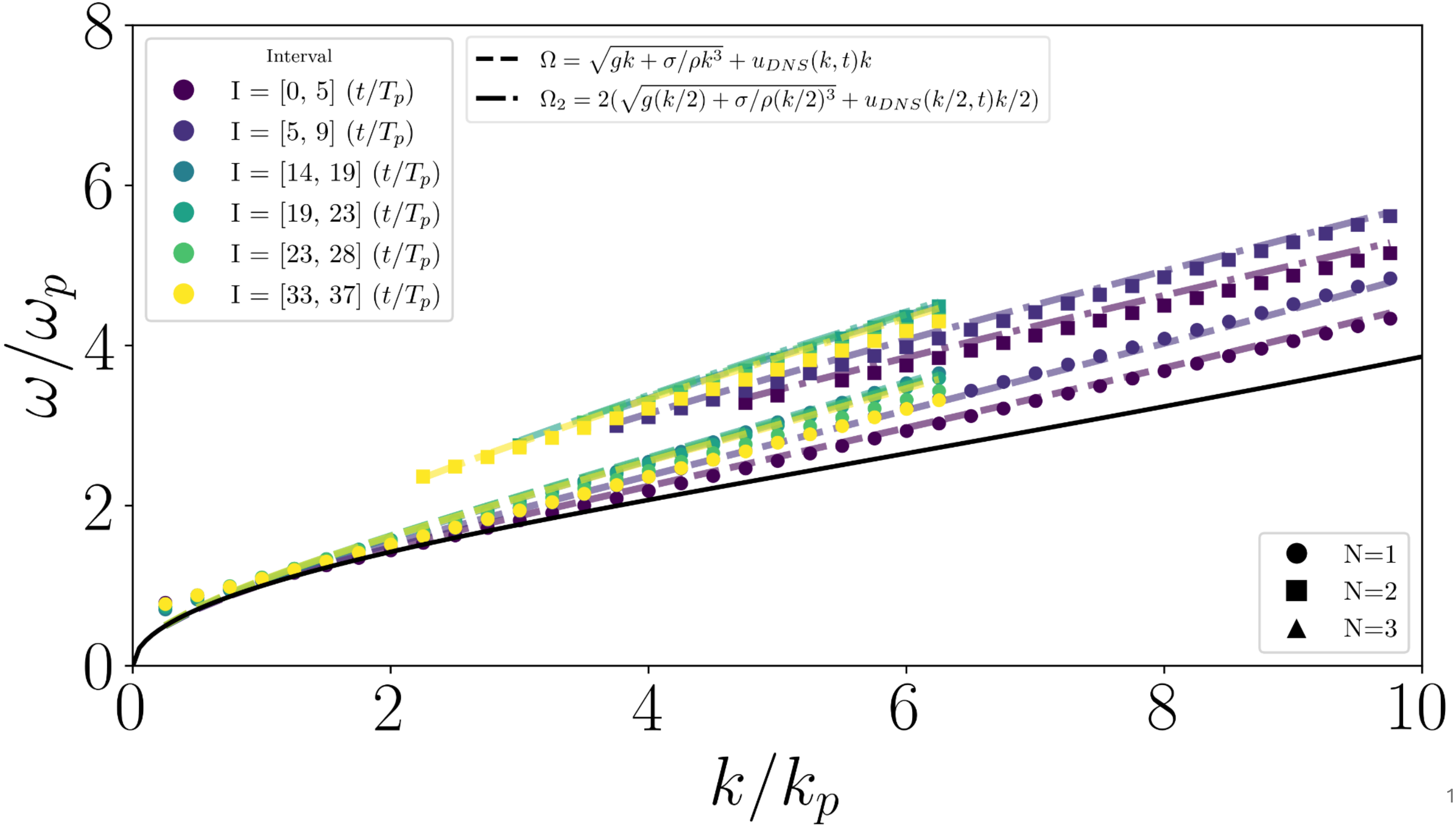
Doppler shift of the first branch from linear dispersion relation



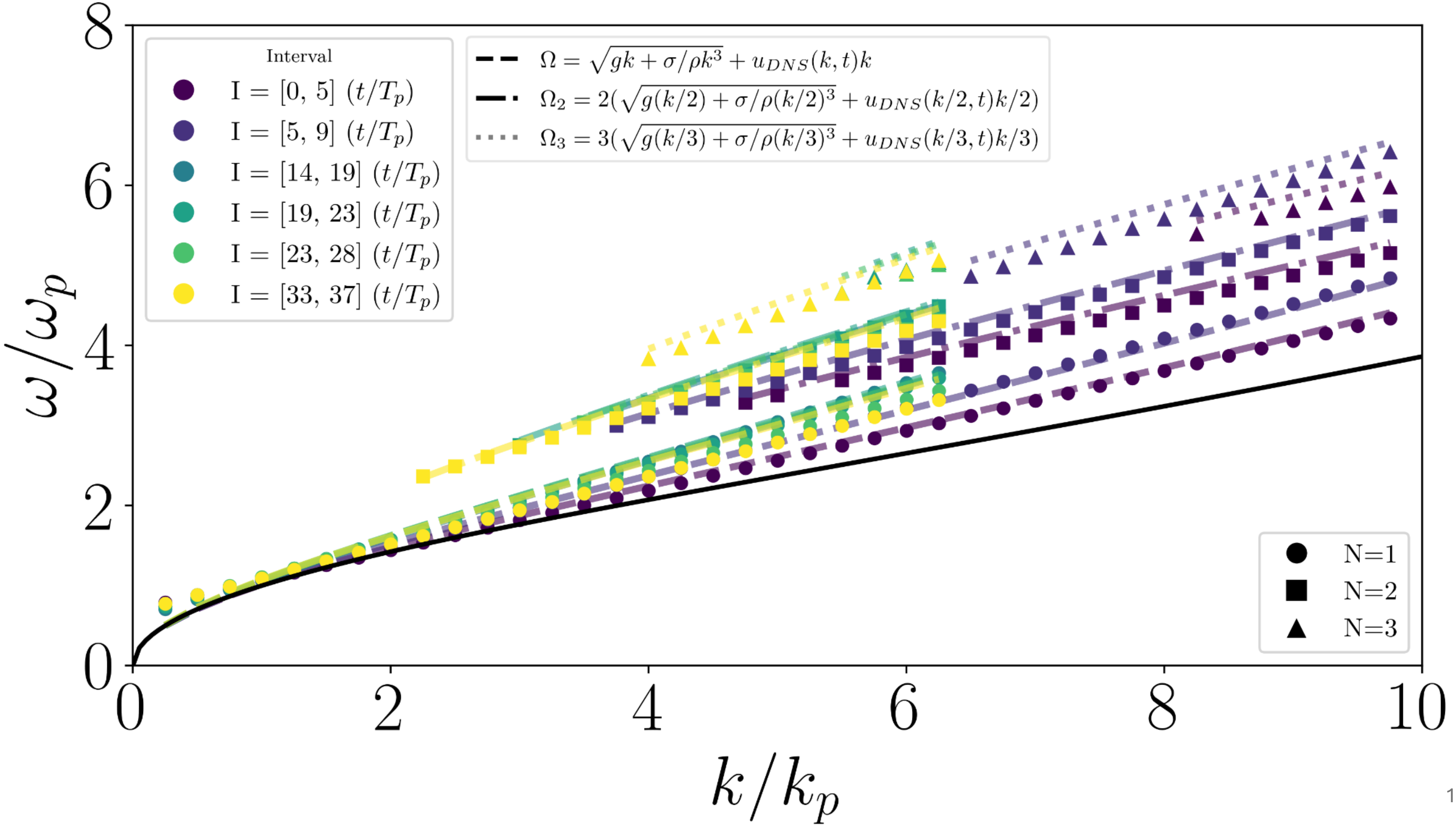
Doppler shift of the first branch from linear dispersion relation



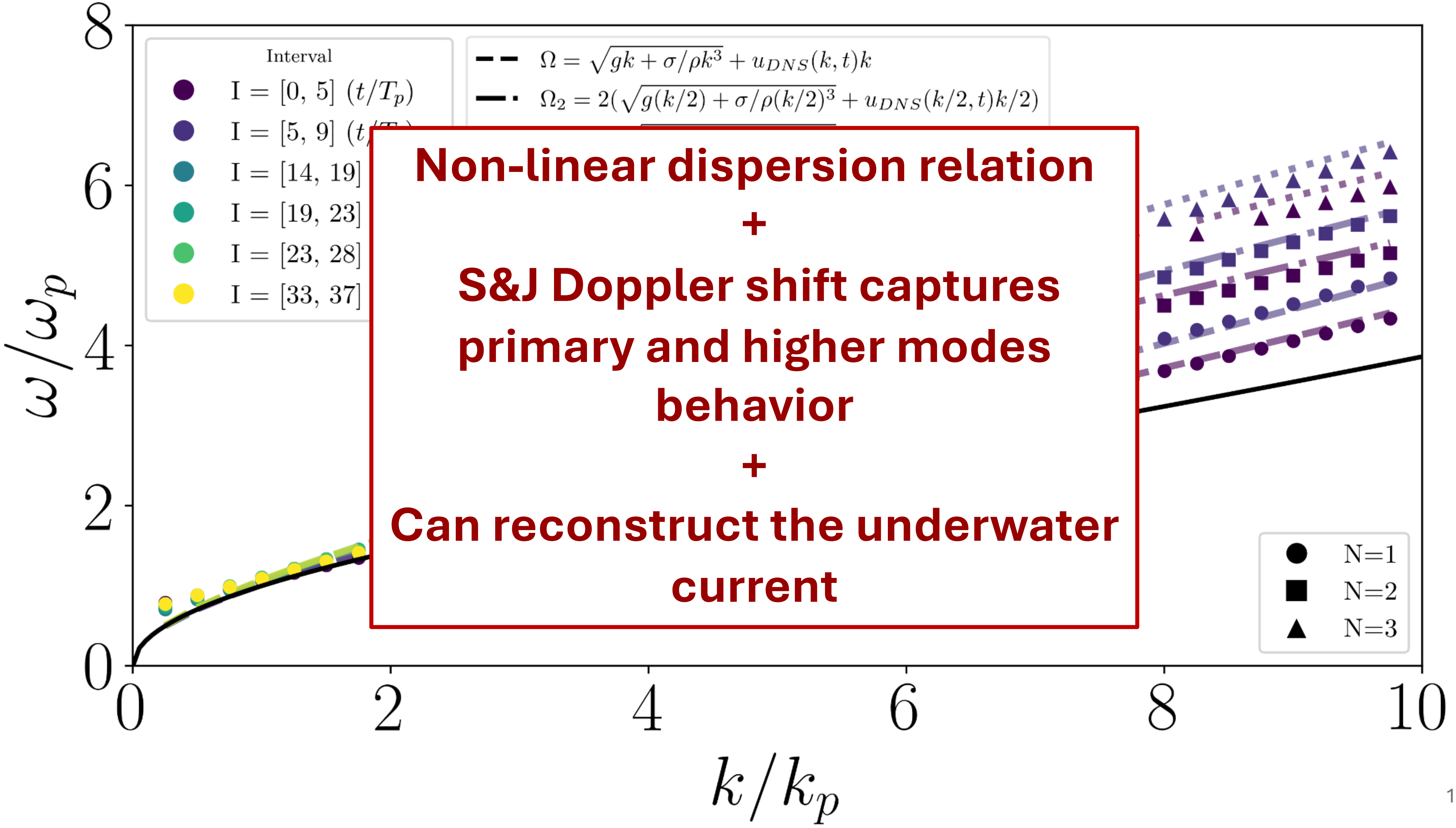
Doppler shift of the other branches following the non-linear dispersion relation



Doppler shift of the other branches following the non-linear dispersion relation



Doppler shift of the other branches following the non-linear dispersion relation



Non-linear dispersion relation

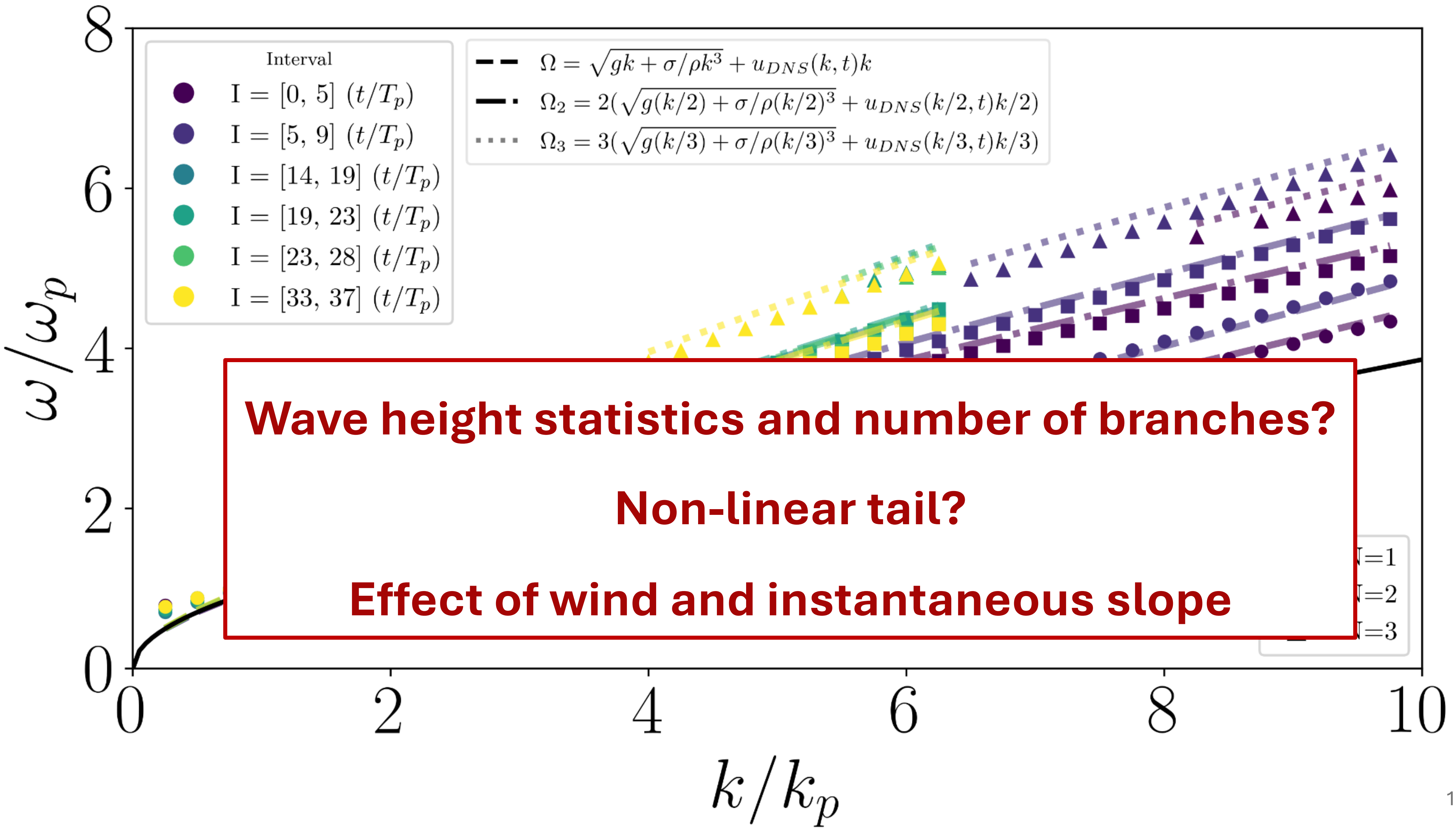
+

S&J Doppler shift captures primary and higher modes behavior

+

Can reconstruct the underwater current

Doppler shift of the other branches following the non-linear dispersion relation

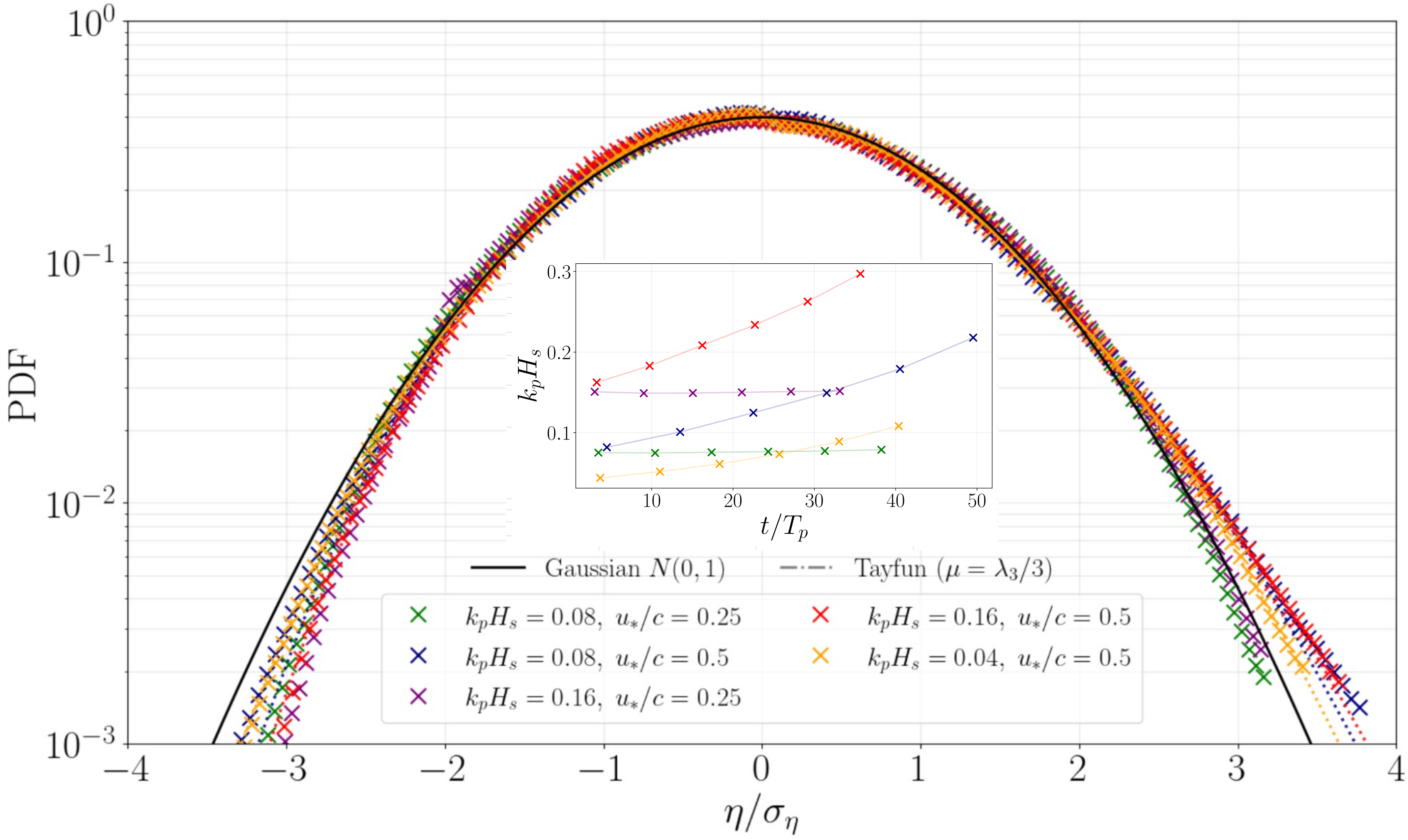


Wave height statistics and number of branches?

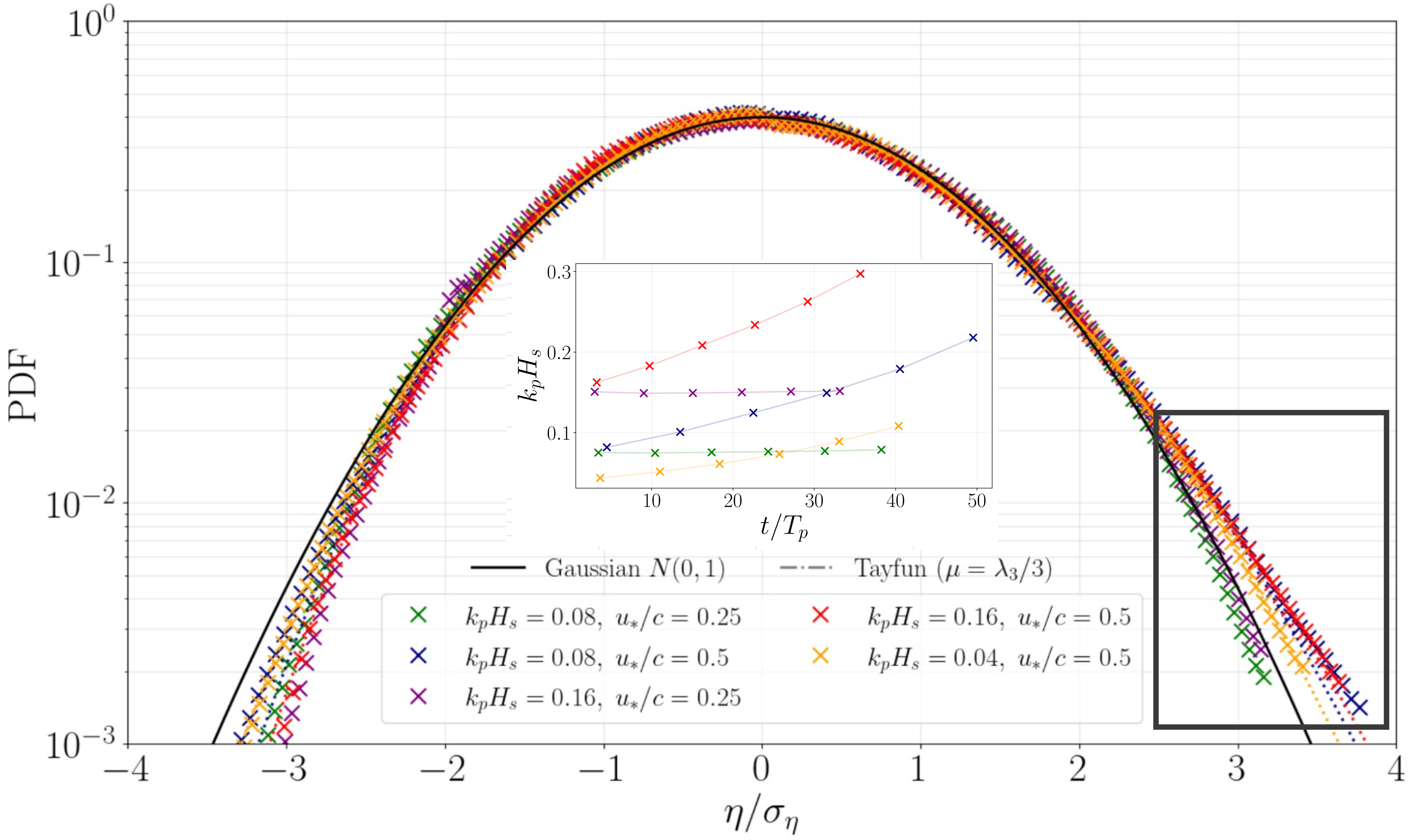
Non-linear tail?

Effect of wind and instantaneous slope

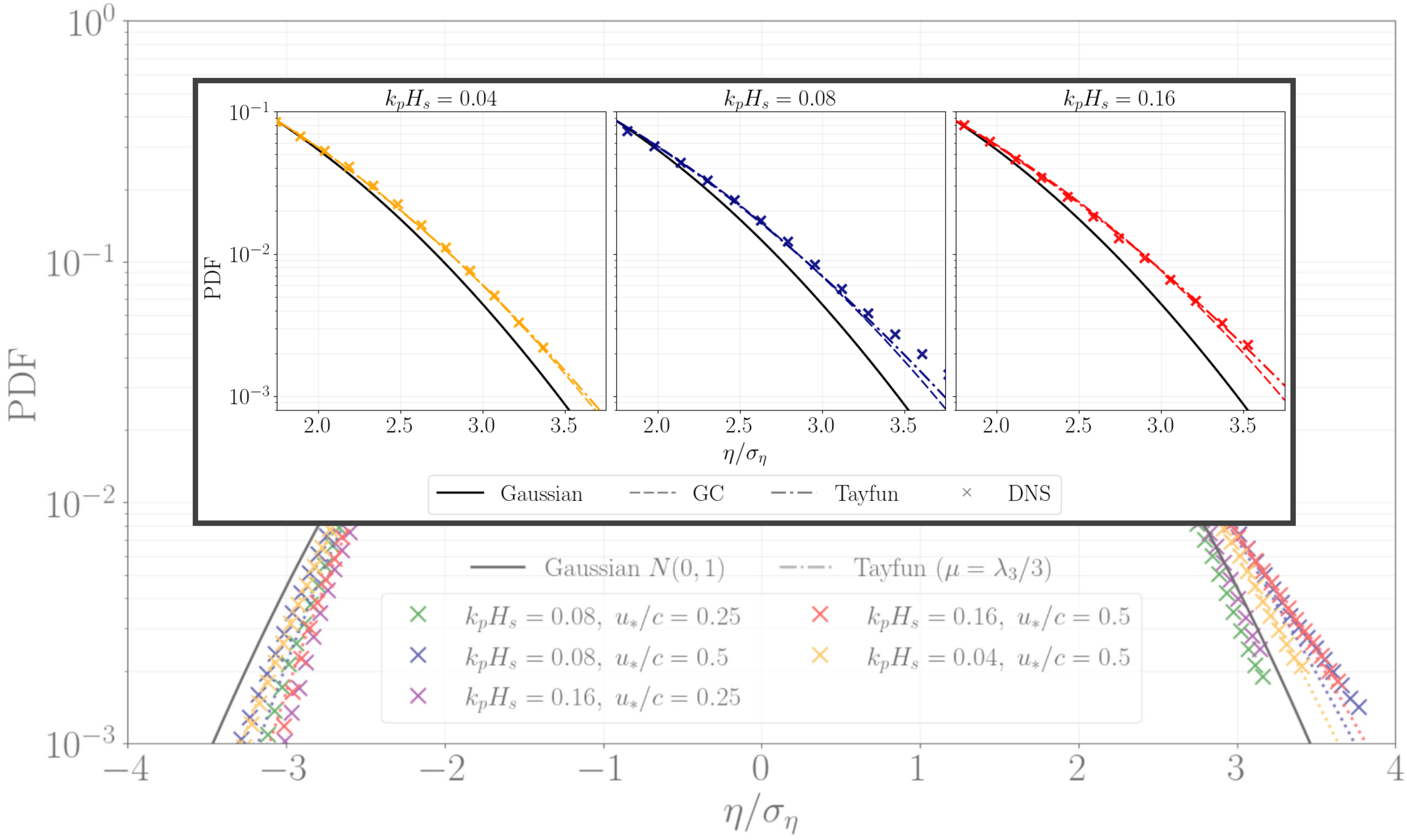
Wave heights statistics



Wave heights statistics



Wave heights statistics



Non-linear waves' propagation and growth across scales

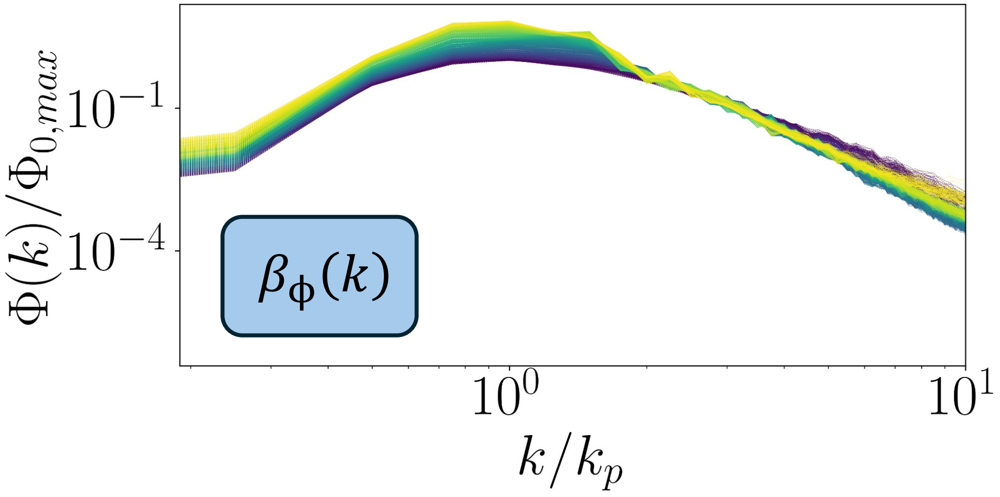
Kinematics of gravity capillary waves above an evolving underwater current

C. Martin Blanco, N. Scapin, J. Wu, S. Popinet, J.T. Farrar, B. Chapron and L. Deike (2026). Journal of Fluid Mechanics.

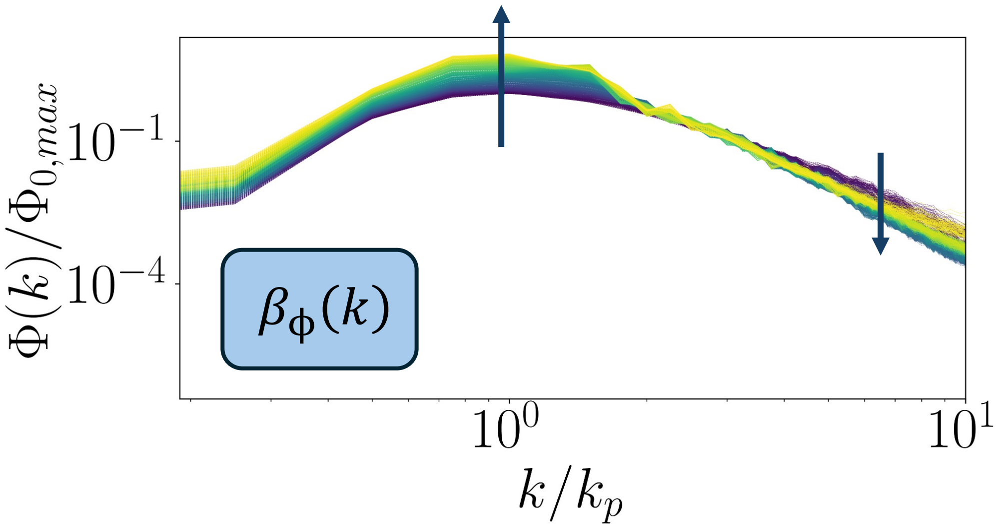
Part II: Dynamics of modes k

How non-linear Doppler shifted waves **grow**?

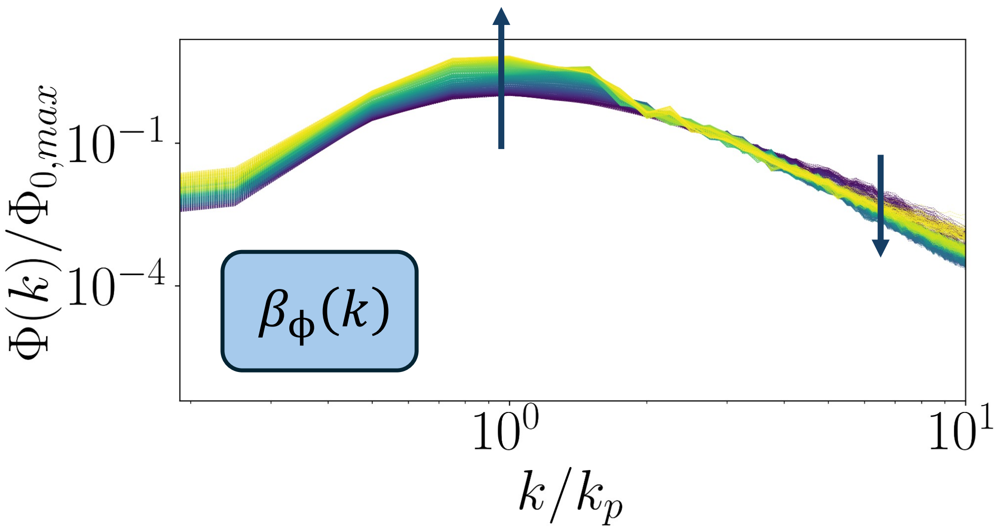
$(k - \omega)$ spectral analysis: growth of modes $\beta_\Phi(k)$ and $\beta_N(k)$

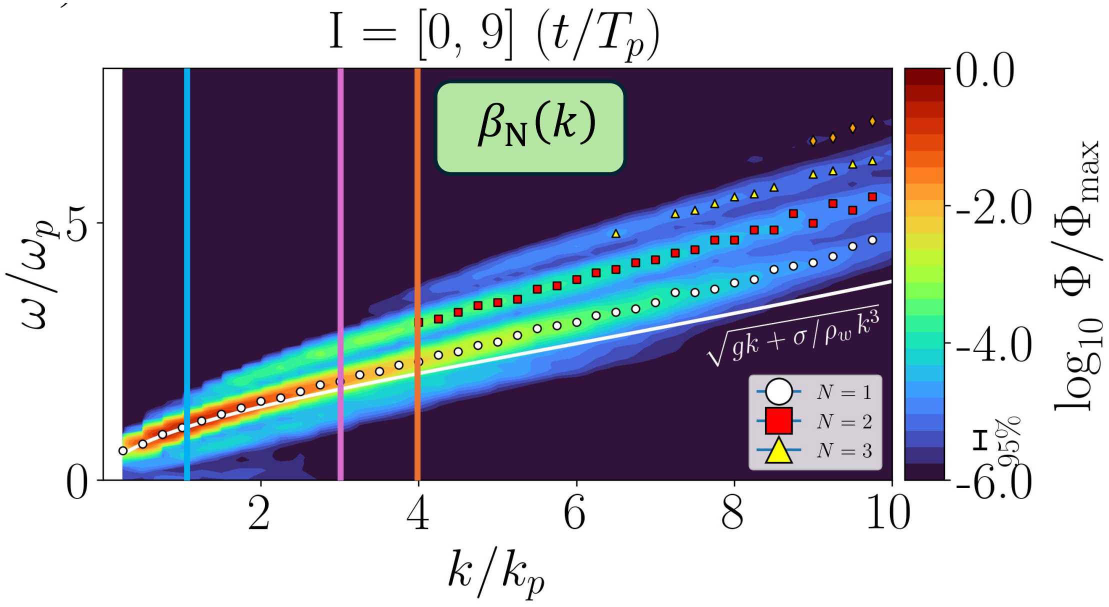


$(k - \omega)$ spectral analysis: growth of modes $\beta_\Phi(k)$ and $\beta_N(k)$

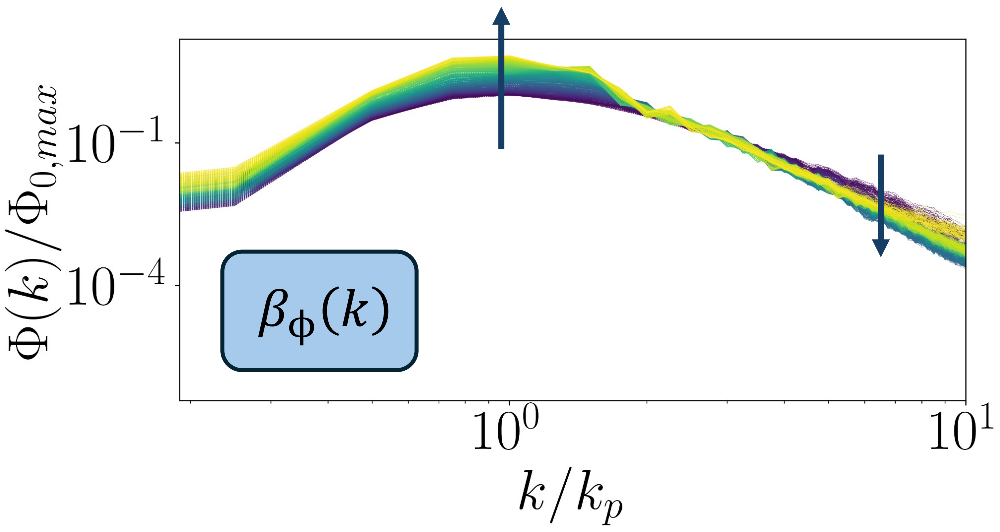


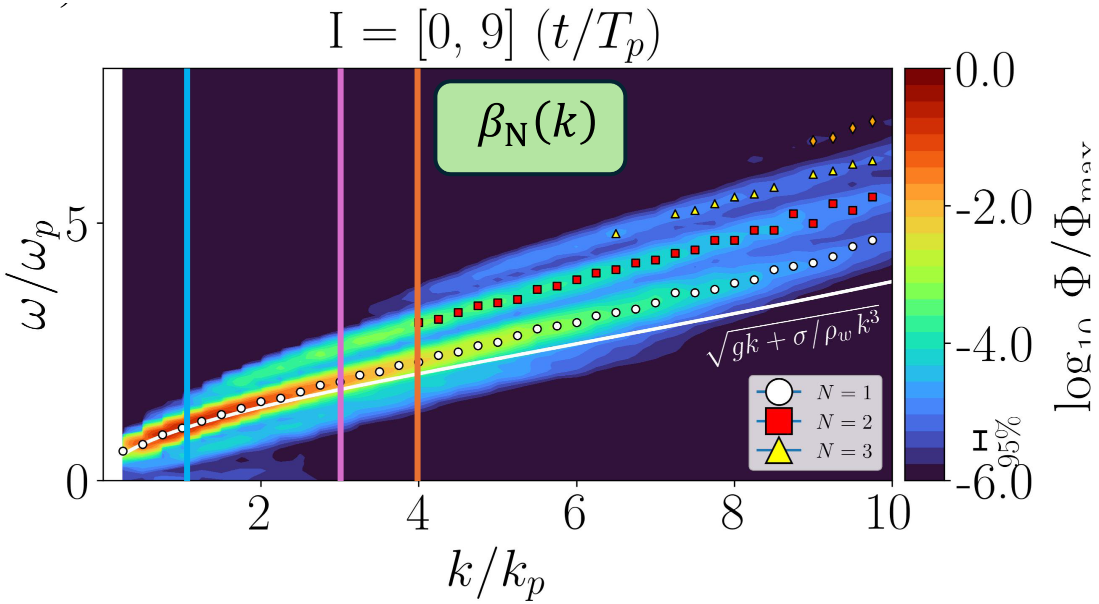
$(k-\omega)$ spectral analysis: total growth rates $\beta_\Phi(k)$ and growth rate per branch $\beta_N(k)$

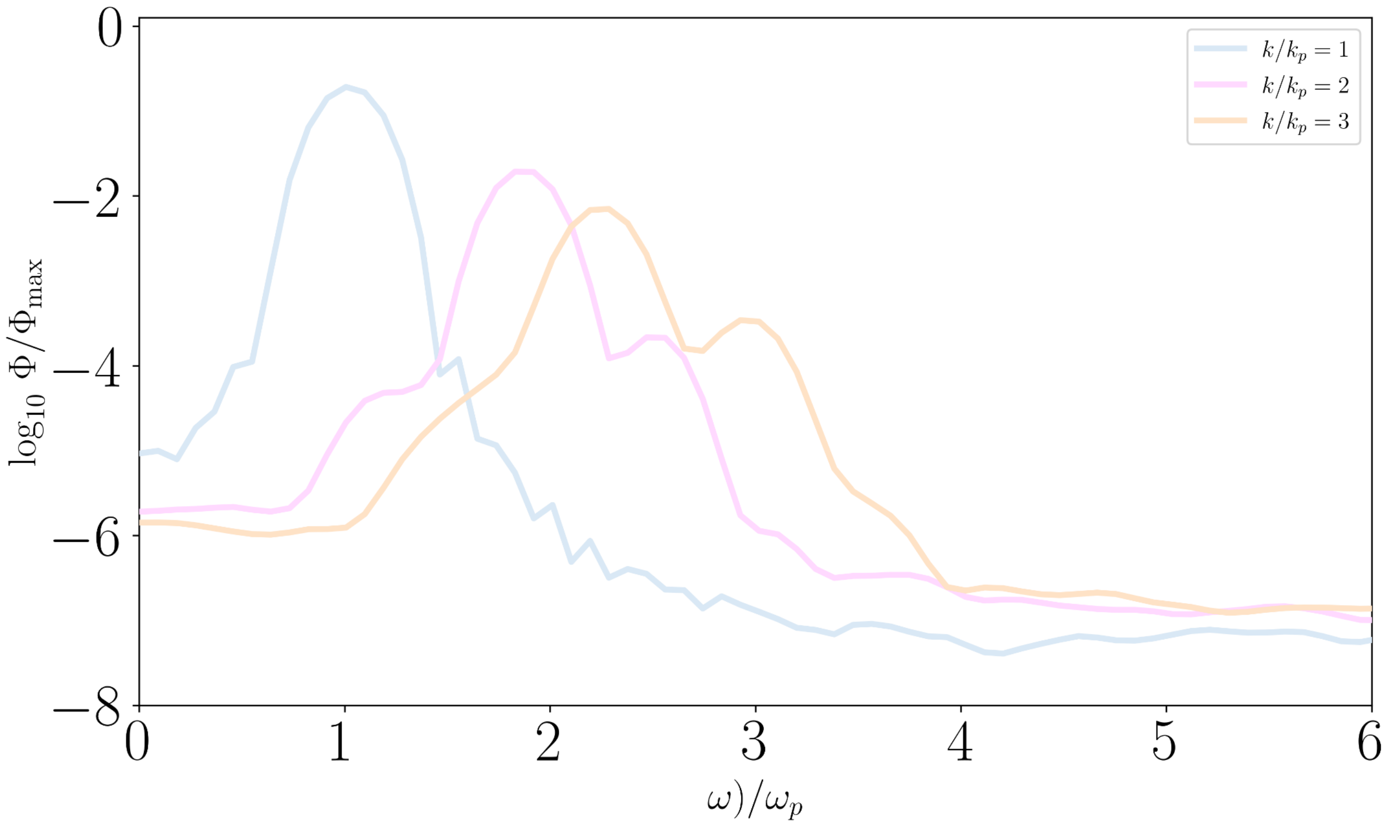




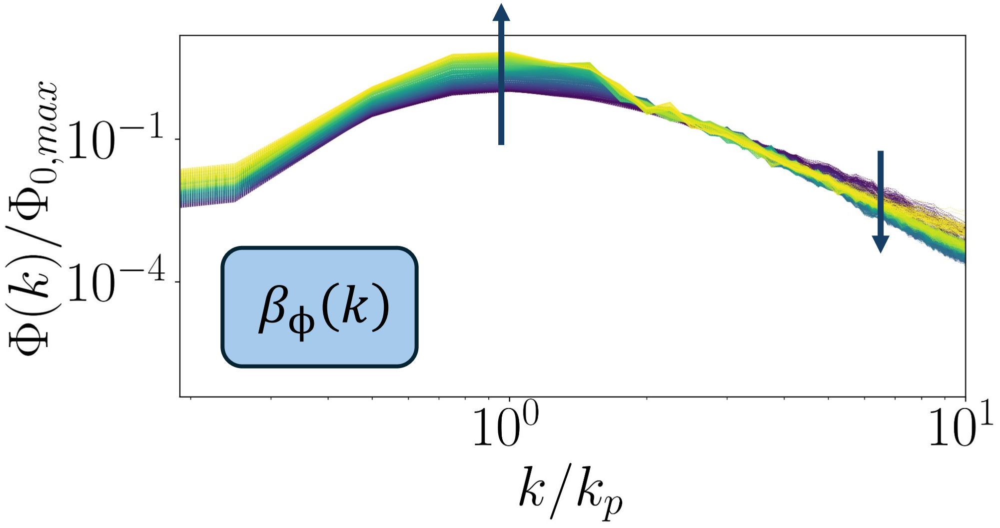
$(k-\omega)$ spectral analysis: total growth rates $\beta_\Phi(k)$ and growth rate per branch $\beta_N(k)$

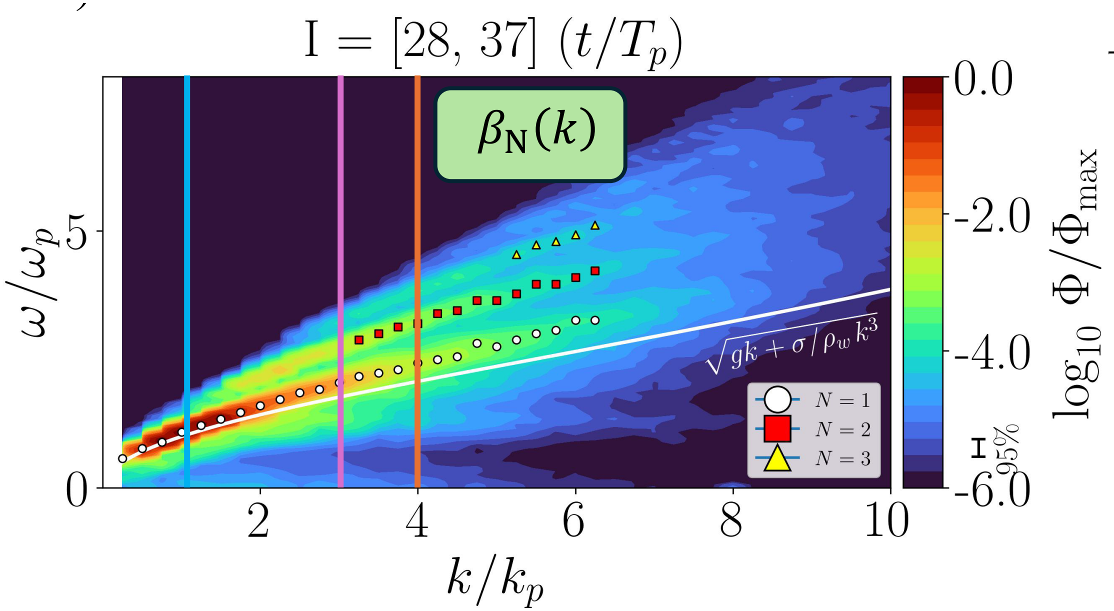


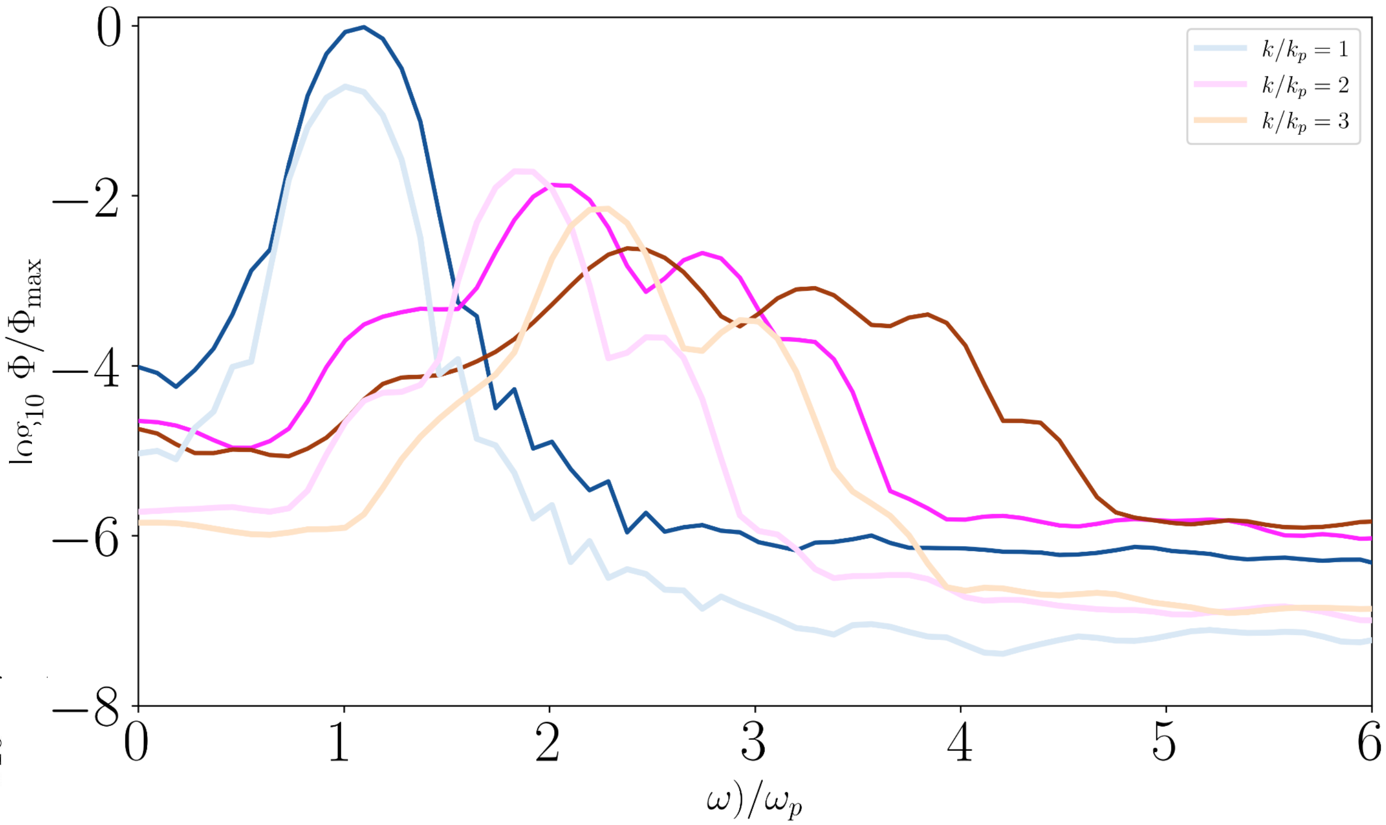




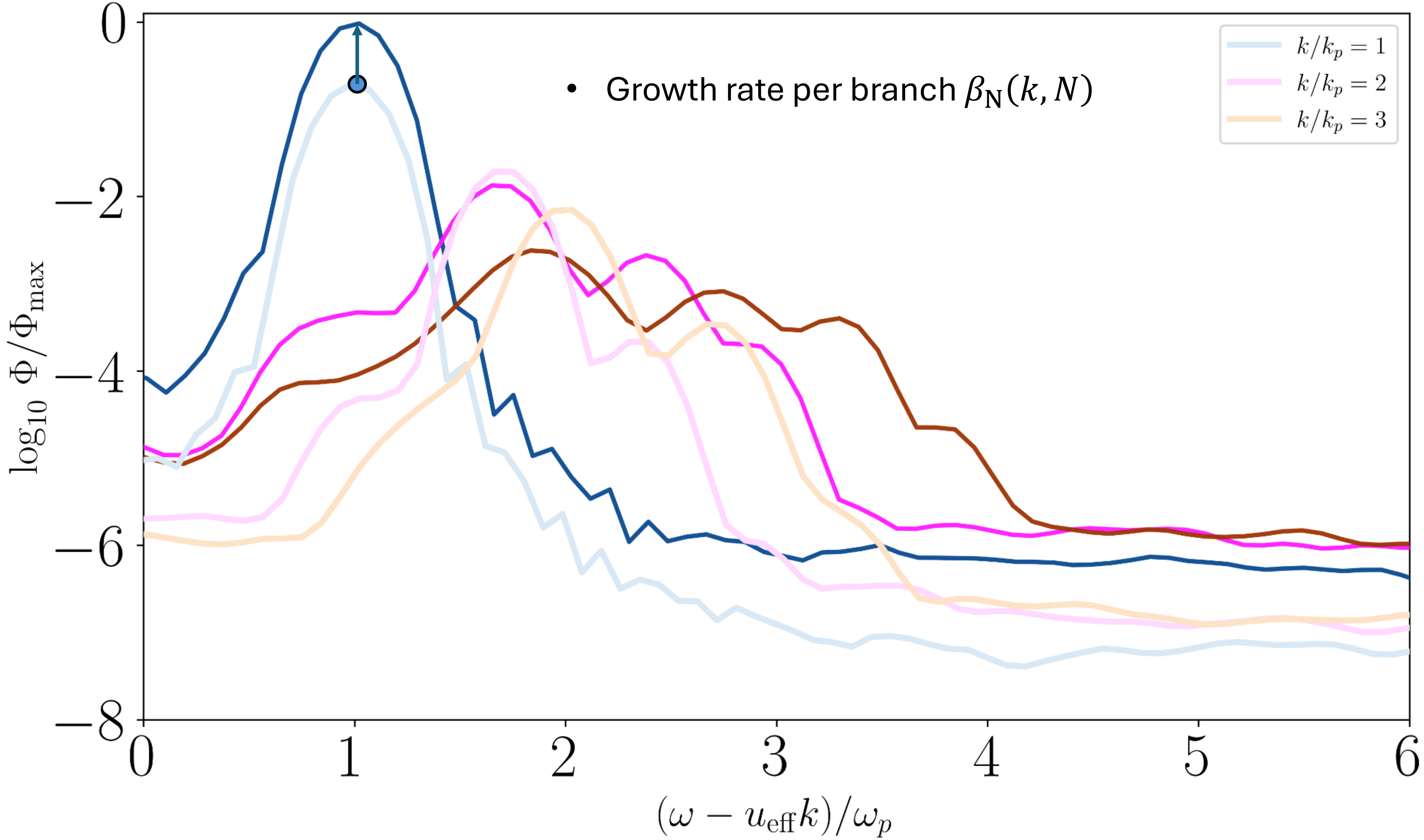
(k − ω) spectral analysis: growth of modes $\beta_\Phi(k)$ and $\beta_N(k)$



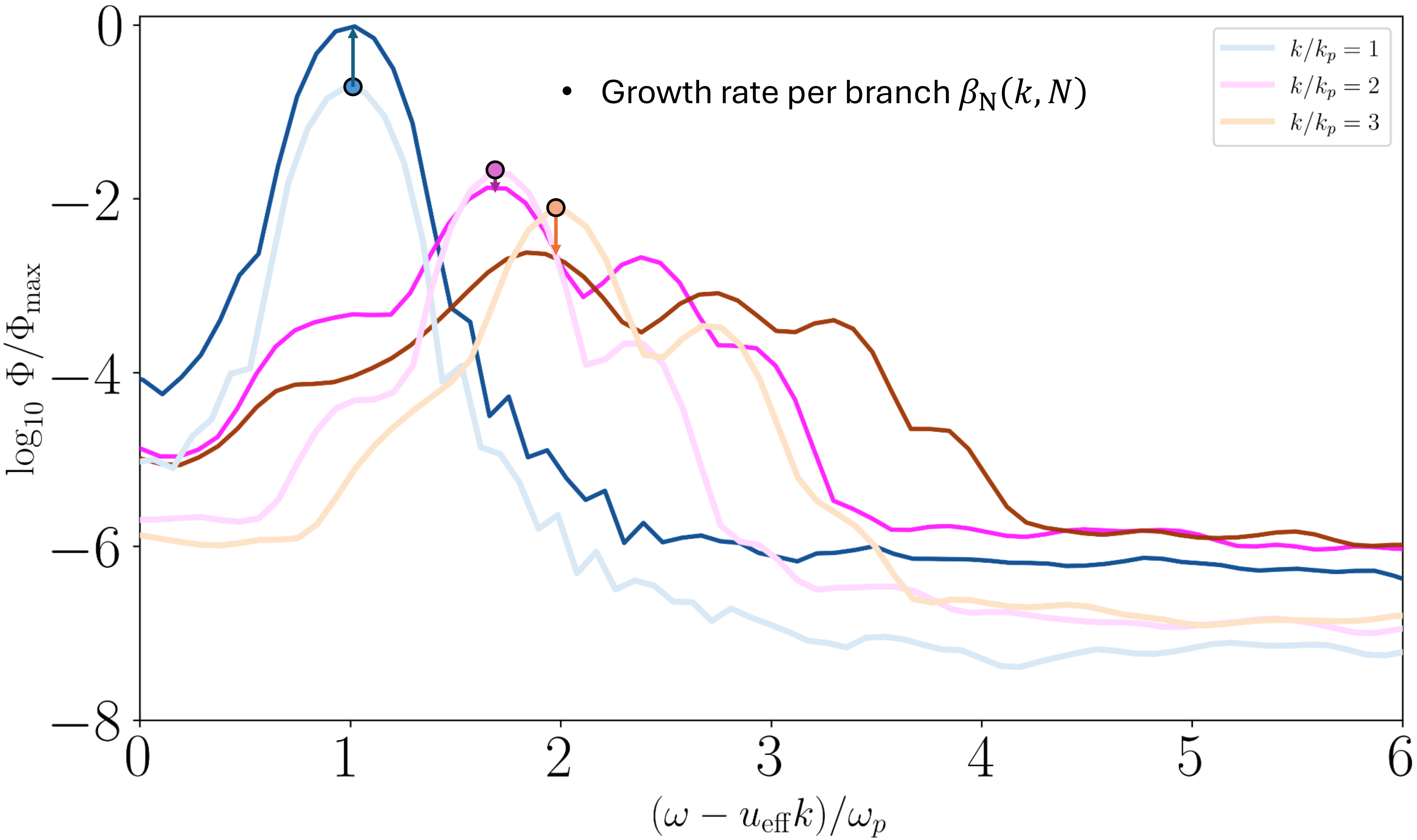




Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$

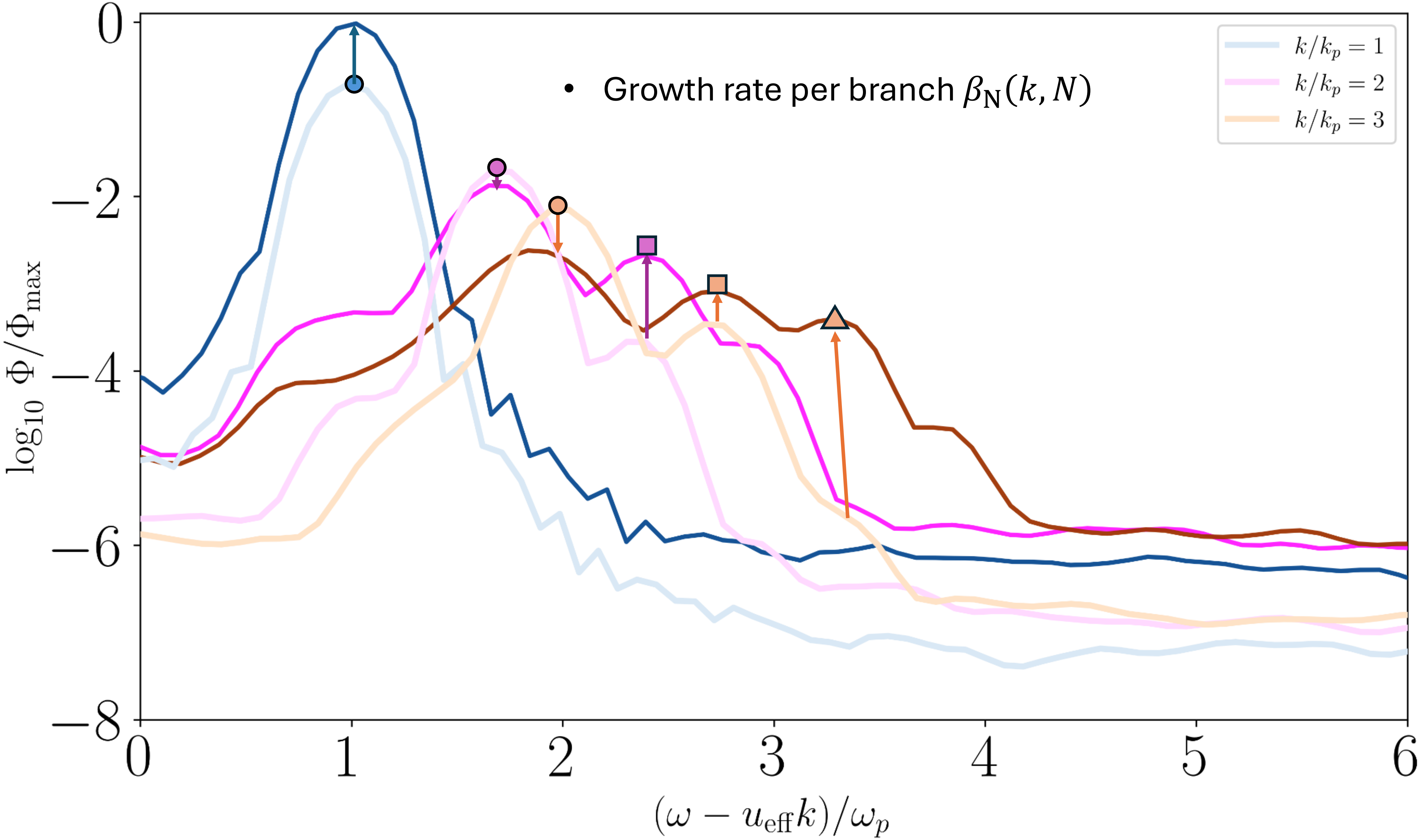


Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$

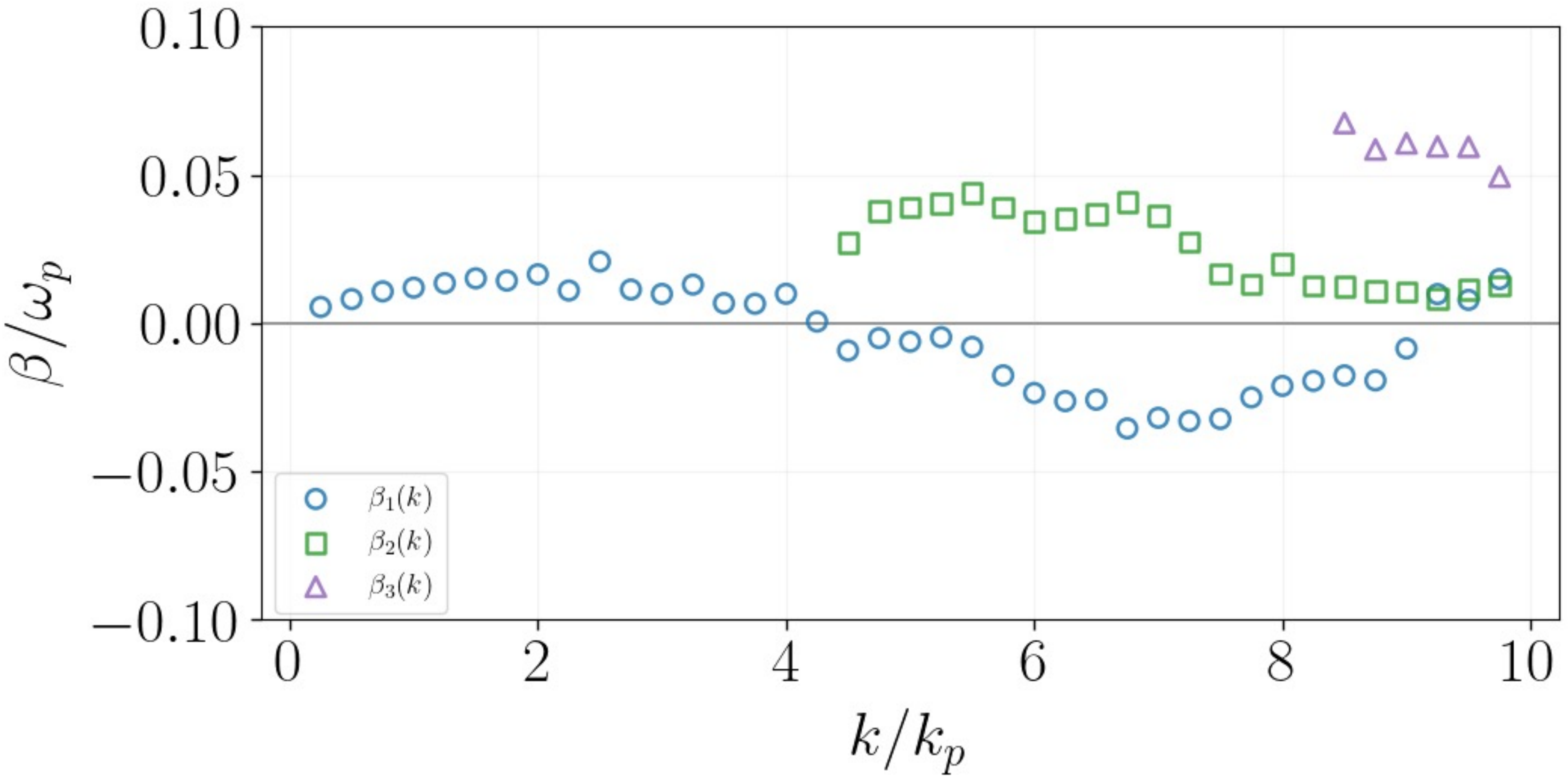


- Growth rate per branch $\beta_N(k, N)$

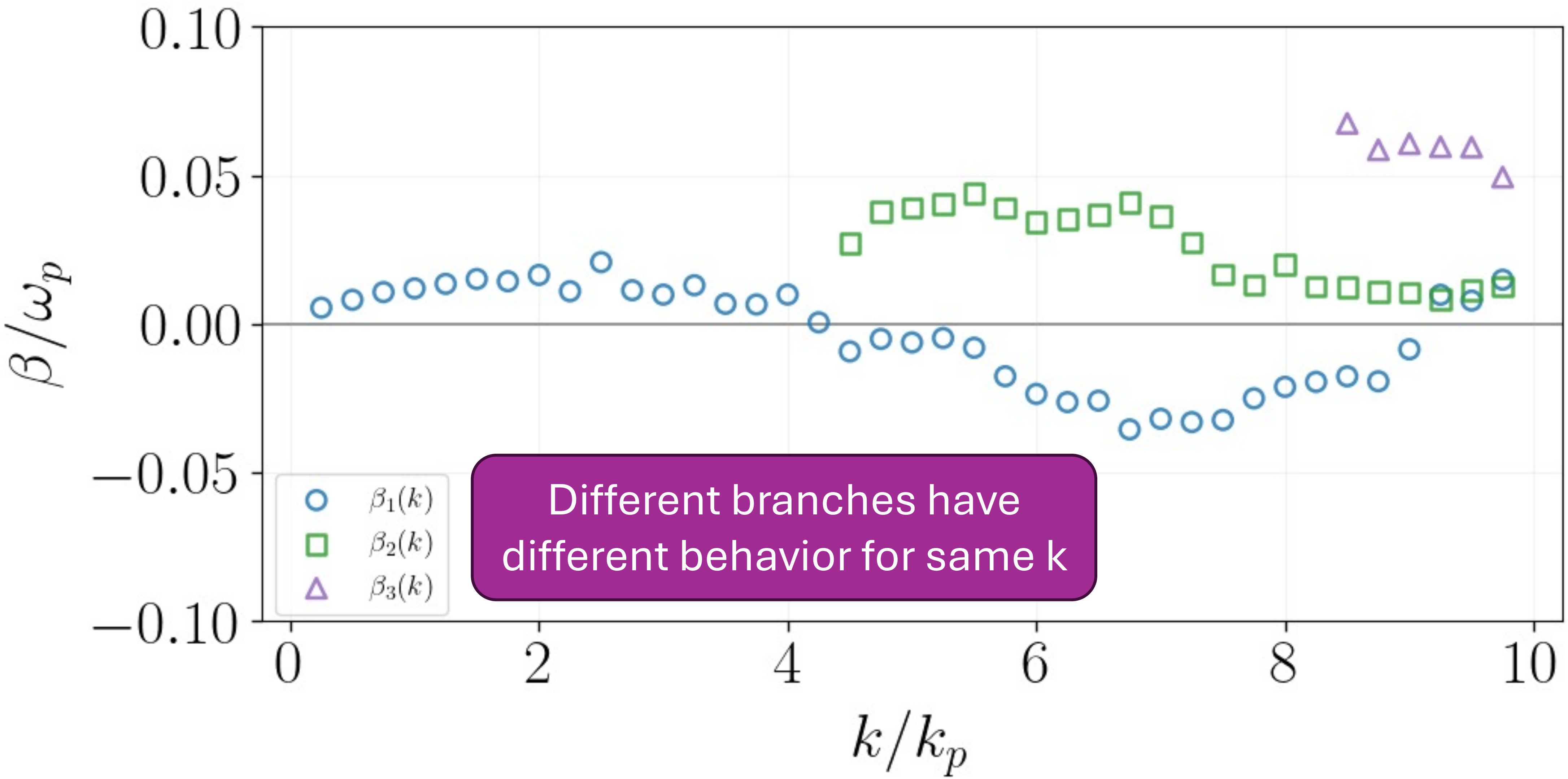
Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$

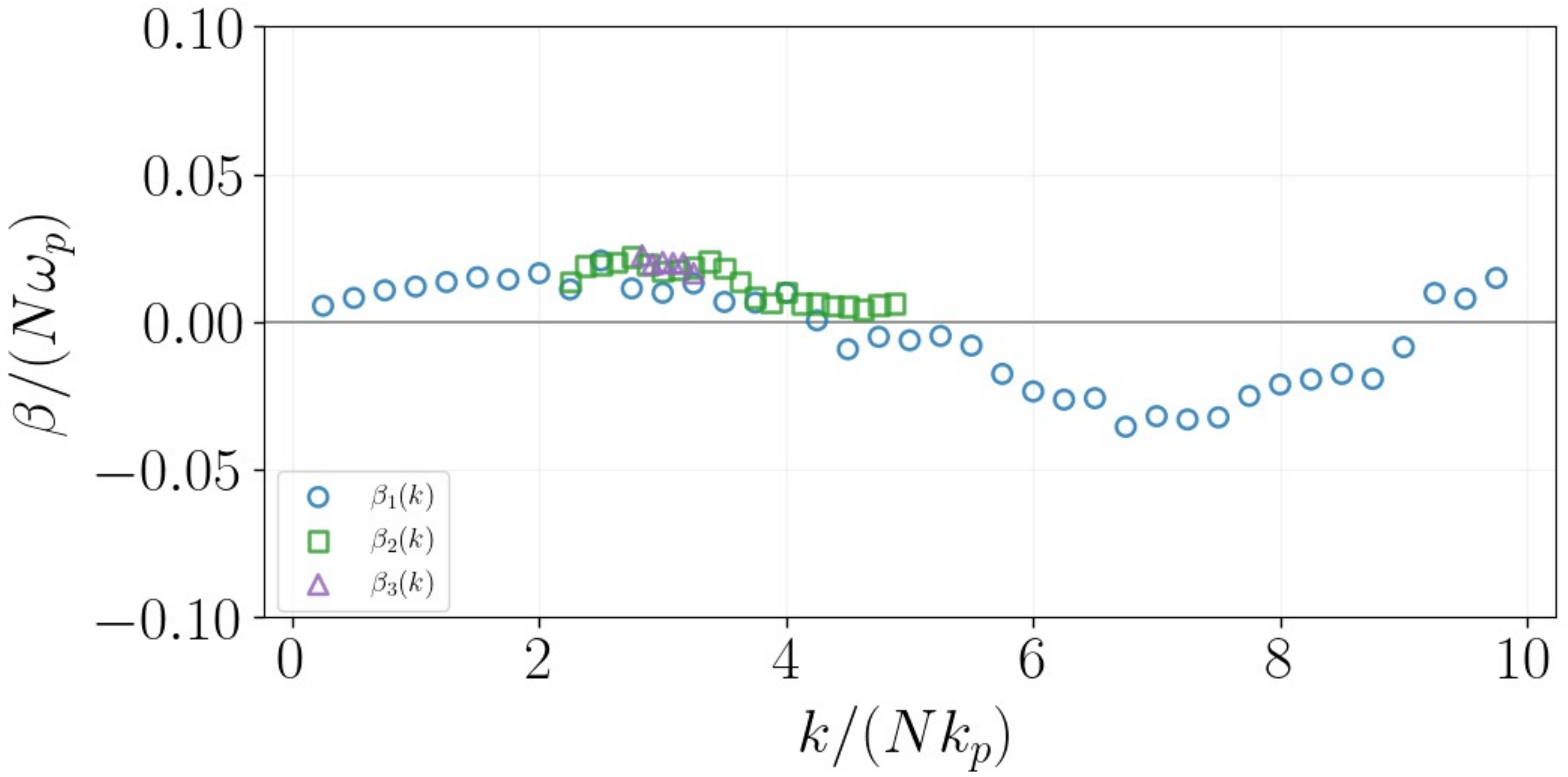


Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$

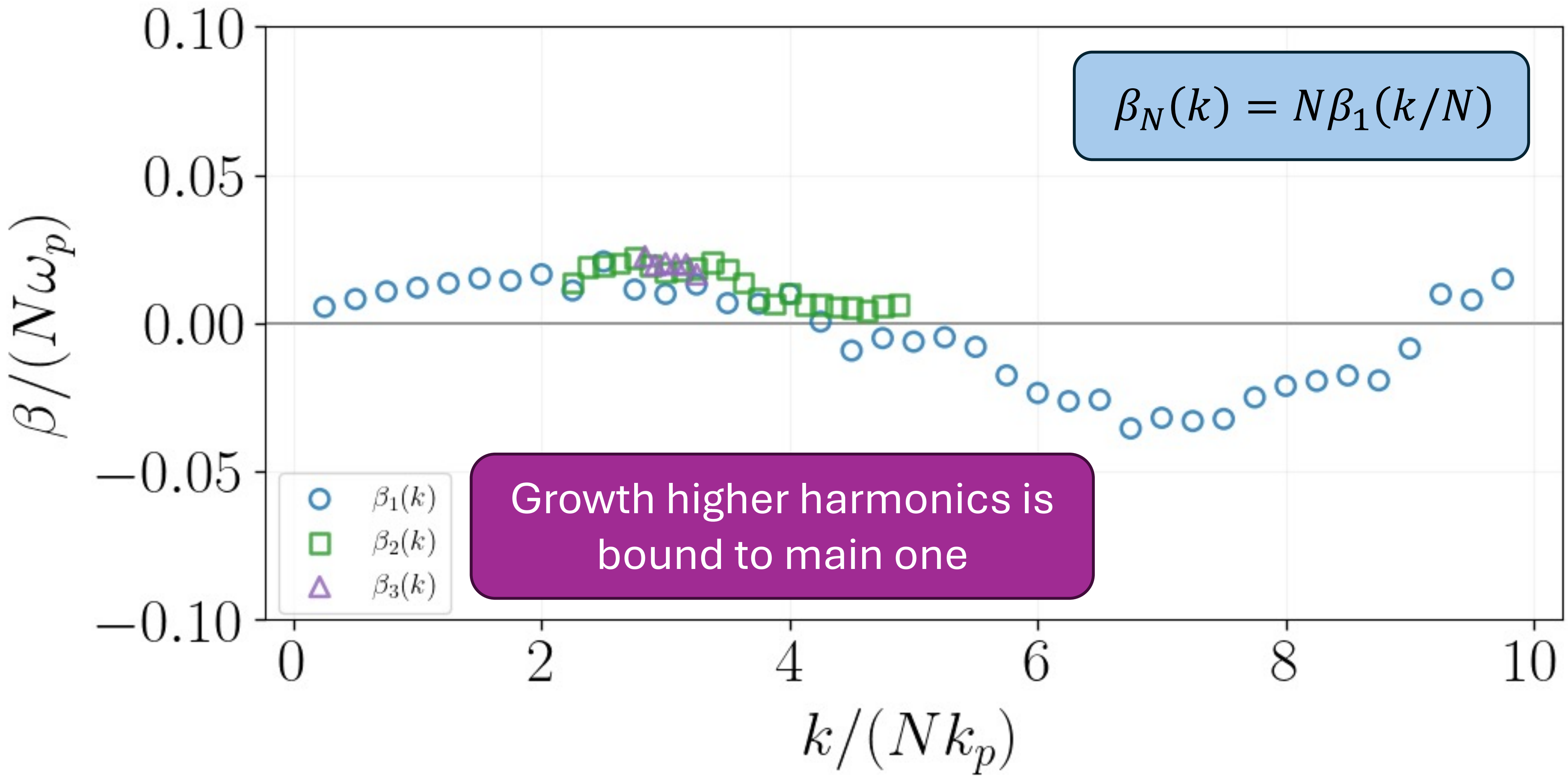


Different branches have different behavior for same k

Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



Growth higher harmonics is bound to main one

Conclusions

- **Bound harmonics** emerge with **increasing wind & wave steepness**, forming distinct nonlinear **dispersion branches**.

- Wave kinematics remain **consistent** with S&J throughout the **transition** of the underwater velocity profile to **turbulence**.

- Wave spectra exhibit significant **broadening** at fixed wavenumber, potentially encoding signatures of **turbulence fluctuations**.

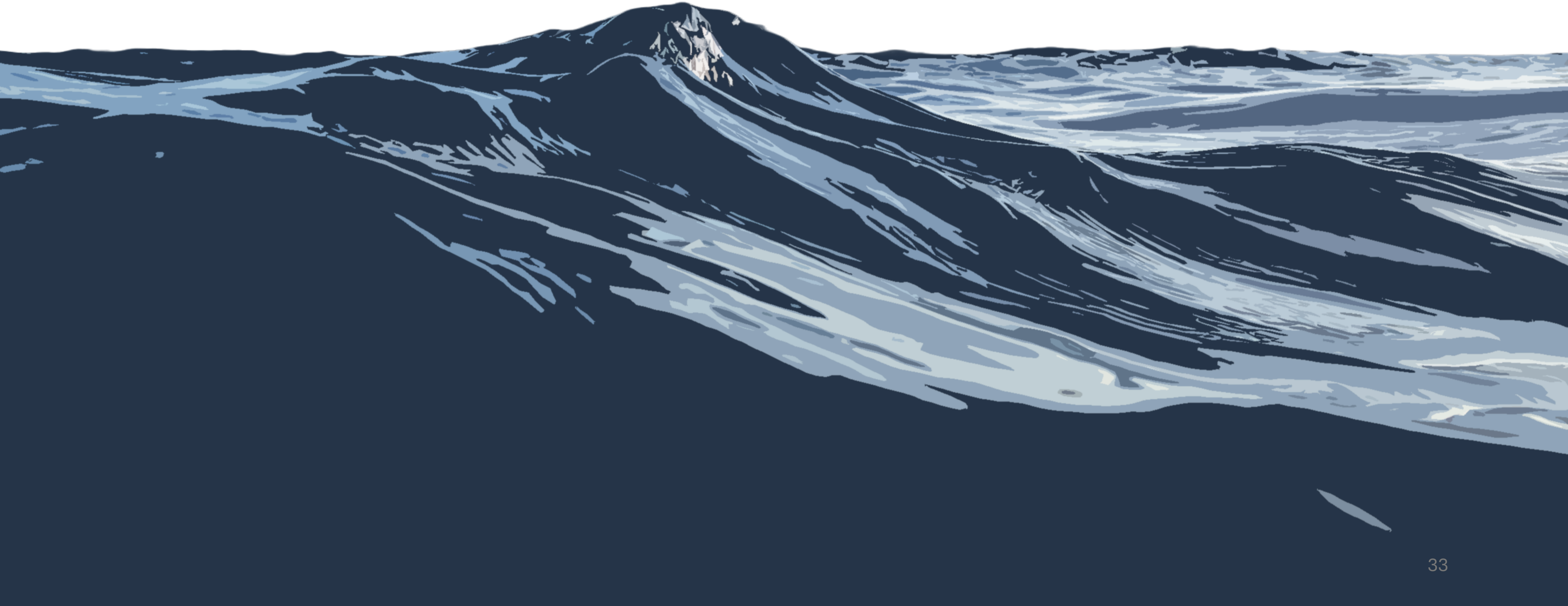
- The subsurface **current profile** can be **obtained** by **inverting** the Doppler-shifted **dispersion relation**.

- The growth rate of higher harmonics can be written as $\beta_N(k) = N\beta_1(k/N)$

- Ongoing quantitative analysis of growth rate and broadening for various conditions

Introduction

Waves **exist** at the **interface** between air and water

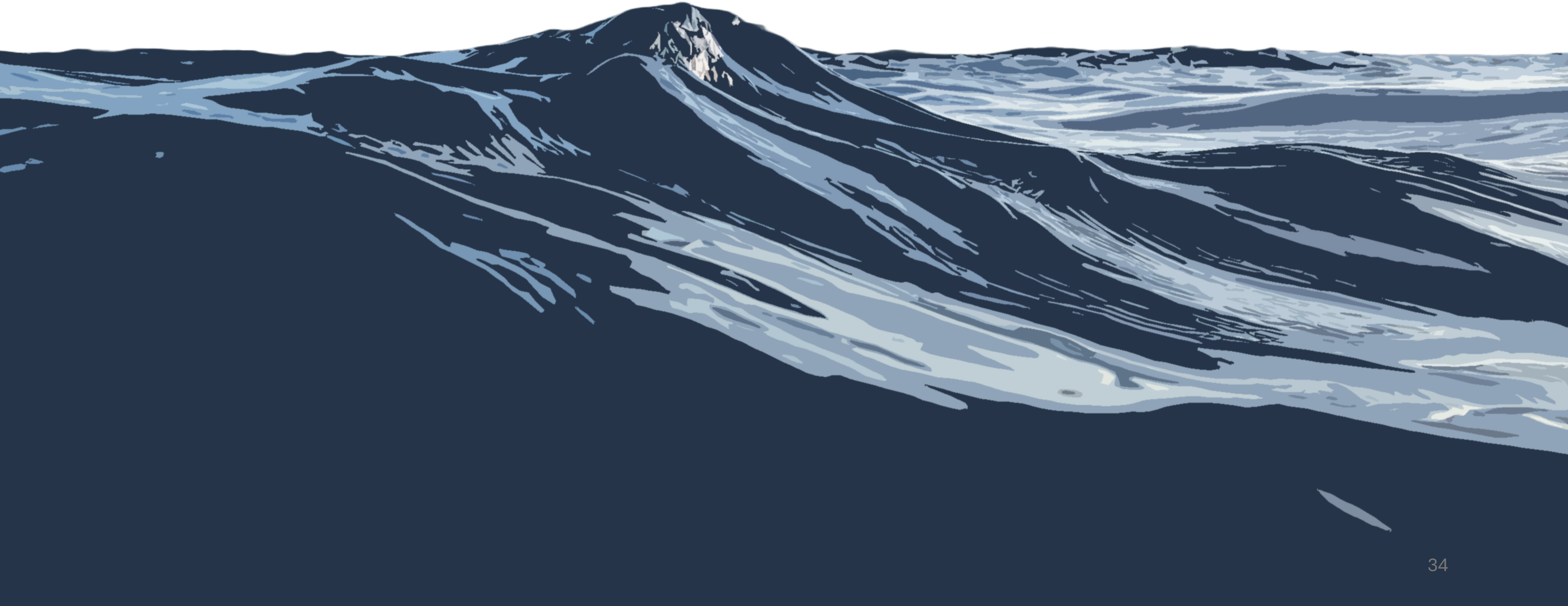


Introduction

Waves **exist** at the **interface** between air and water

Atmospheric Boundary Layer

Momentum

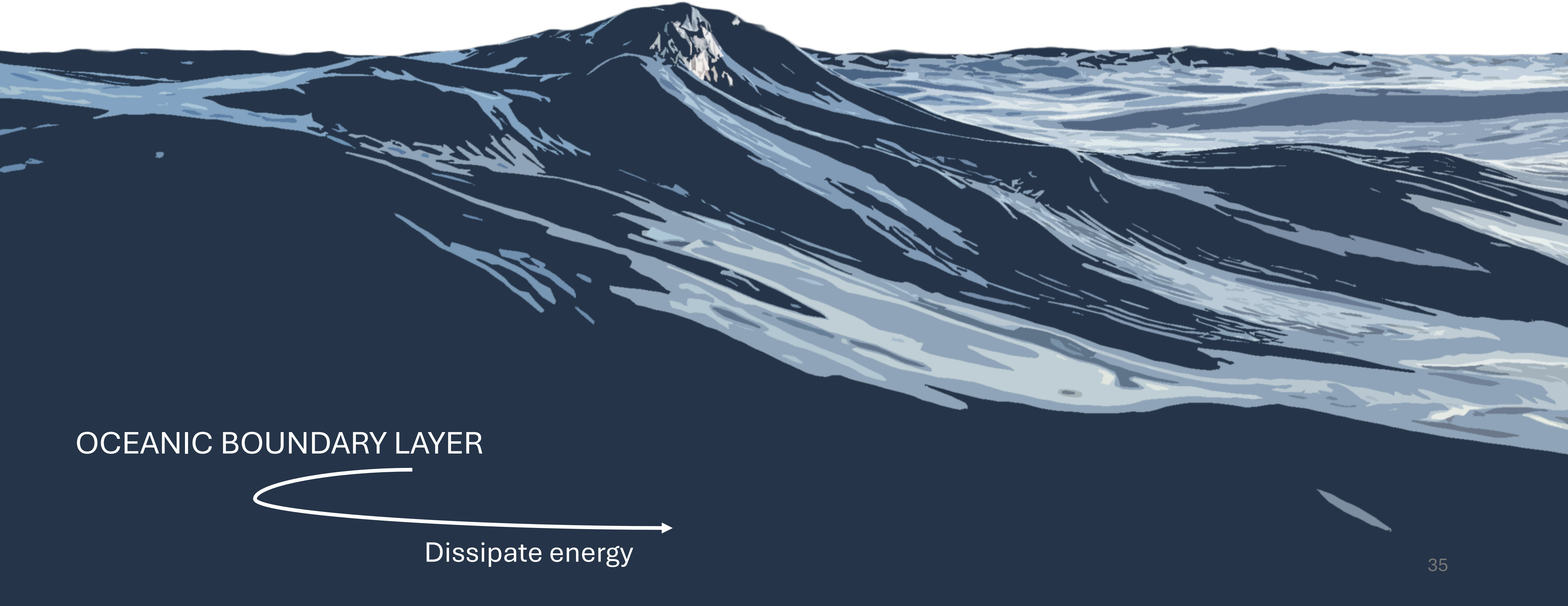


Introduction

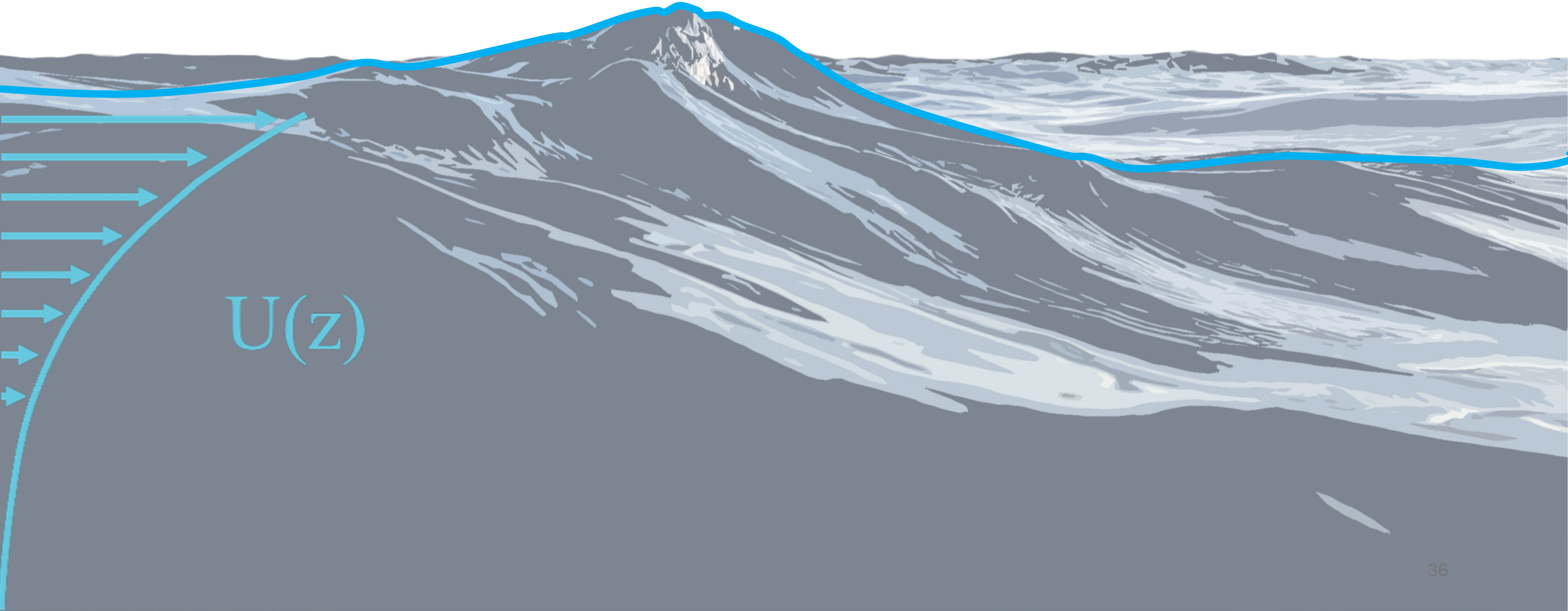
Waves **exist** at the **interface** between air and water

Atmospheric Boundary Layer

Momentum



Introduction

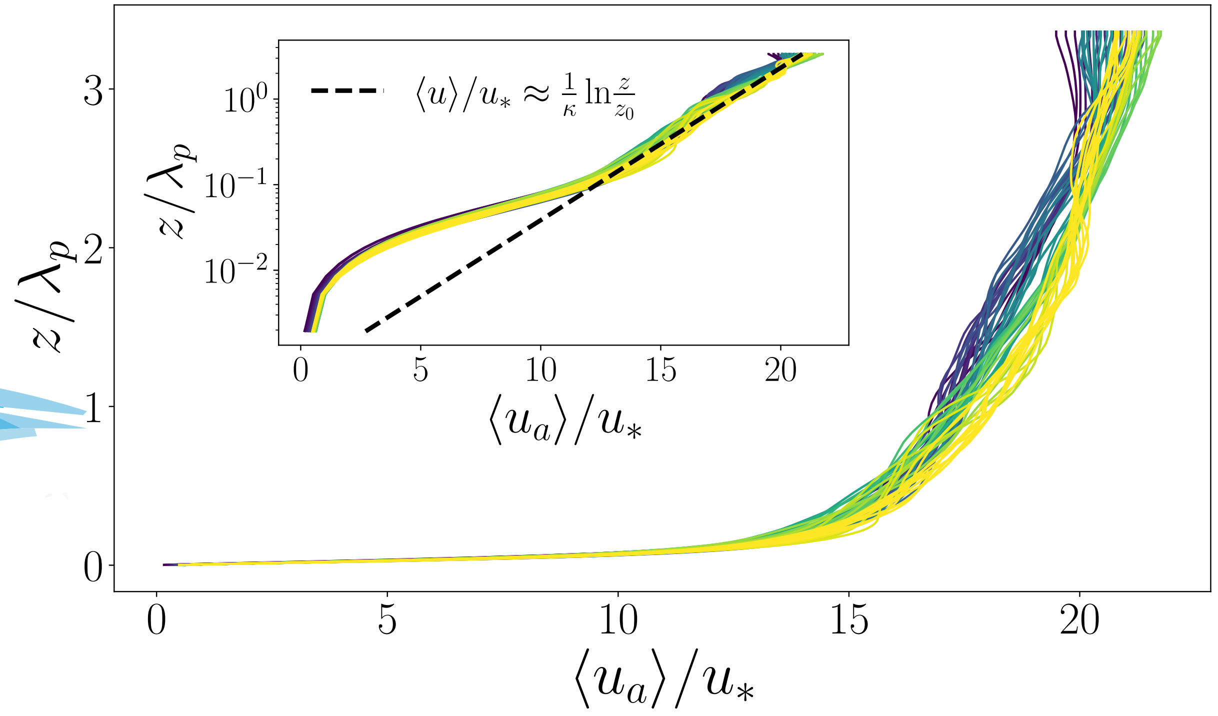


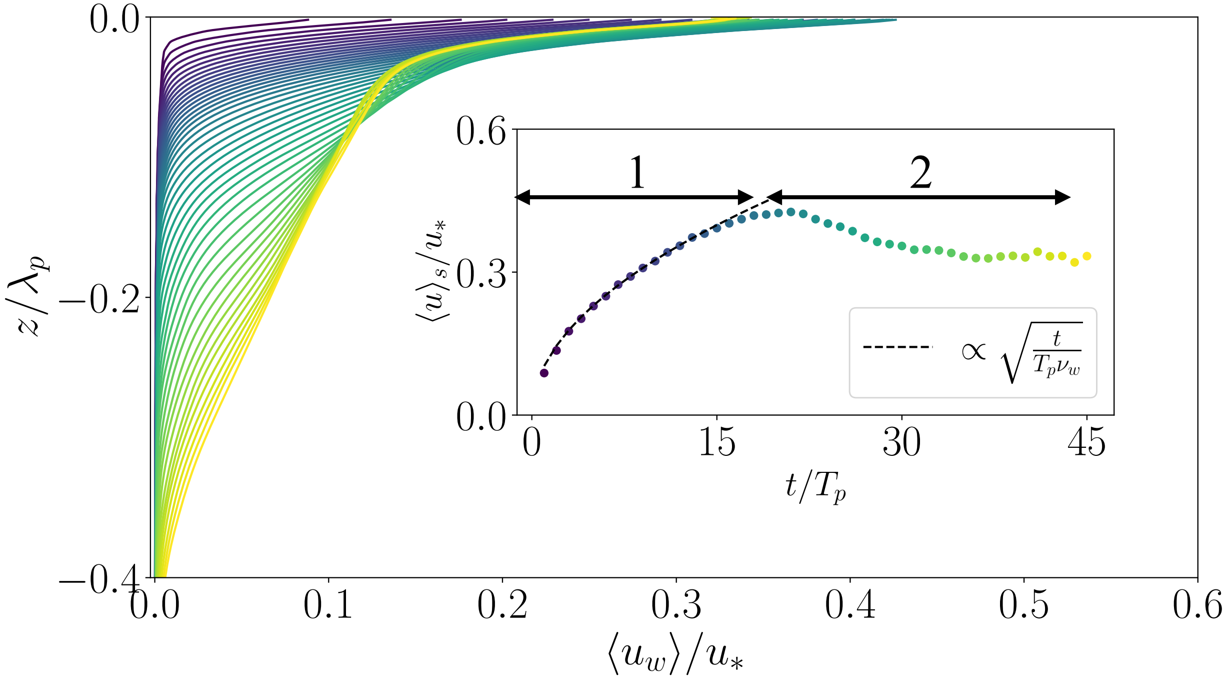
Mean Flow: Mean velocity profile in the **air** and the **water**

$k_p H_s = 0.16$

$u_*/c = 0.5$

Air-side is **statistically stationary**

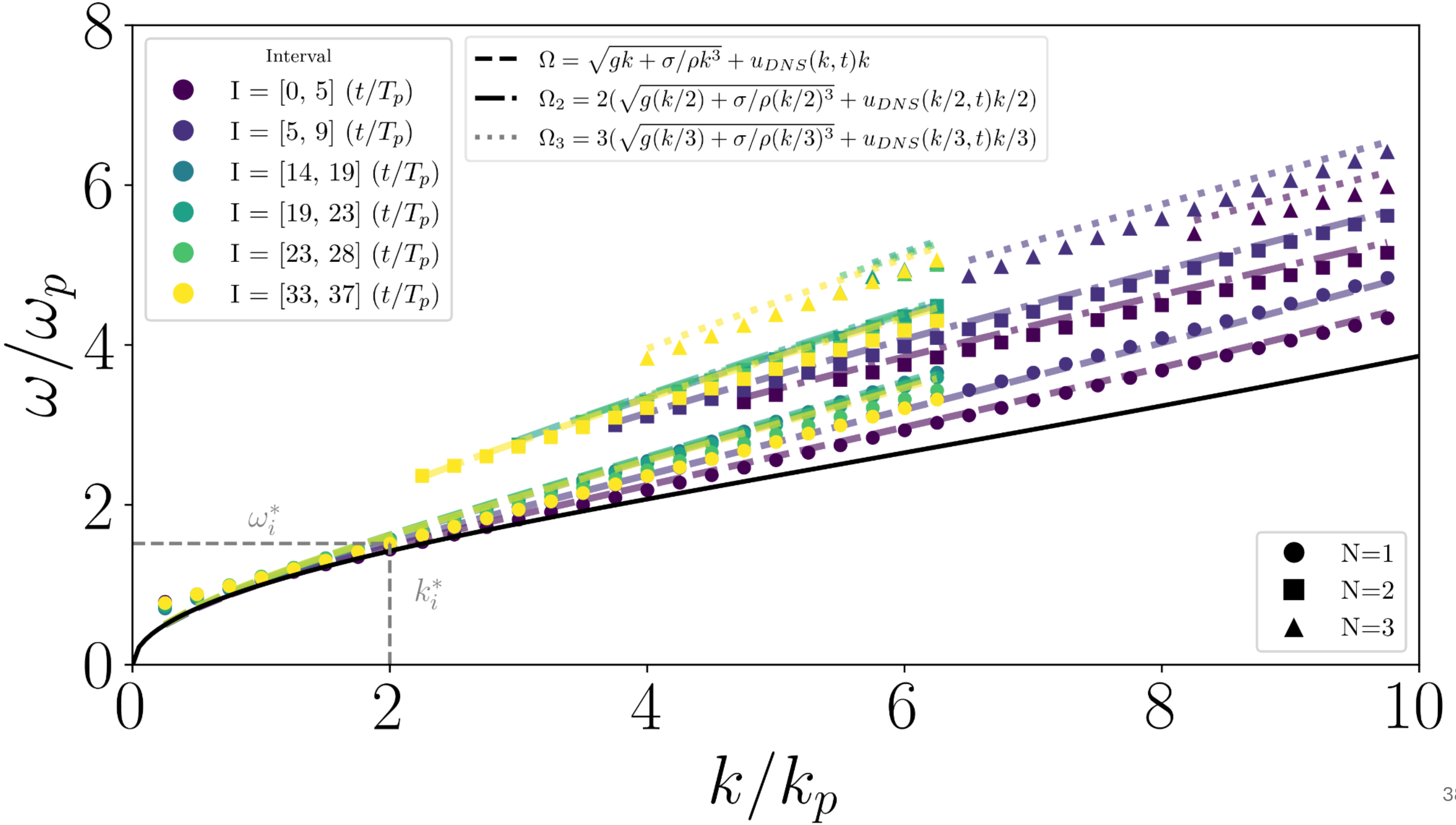




Developing viscous layer that transitions to a **turbulent boundary layer**

1. **Viscous** momentum diffusion
2. Fully **developed turbulence**

Phase speed of the higher bound modes



Branches Time Evolution: Higher harmonics and non-linear dispersion relation

Primary mode

(k_*, ω_*) ⟶ **Non-linear** interaction with itself

Higher harmonics

$$\left.\begin{matrix}(2k_*, 2\omega_*)\\ (3k_*, 3\omega_*)\end{matrix}\right|\begin{matrix}k_N = Nk_*\\ \Omega_N = N\omega_*\end{matrix}$$

↓

$$\Omega_N(k_N) = N\sqrt{\frac{gk_N}{N} + \frac{\sigma}{\rho}\left(\frac{k_N}{N}\right)^3}$$

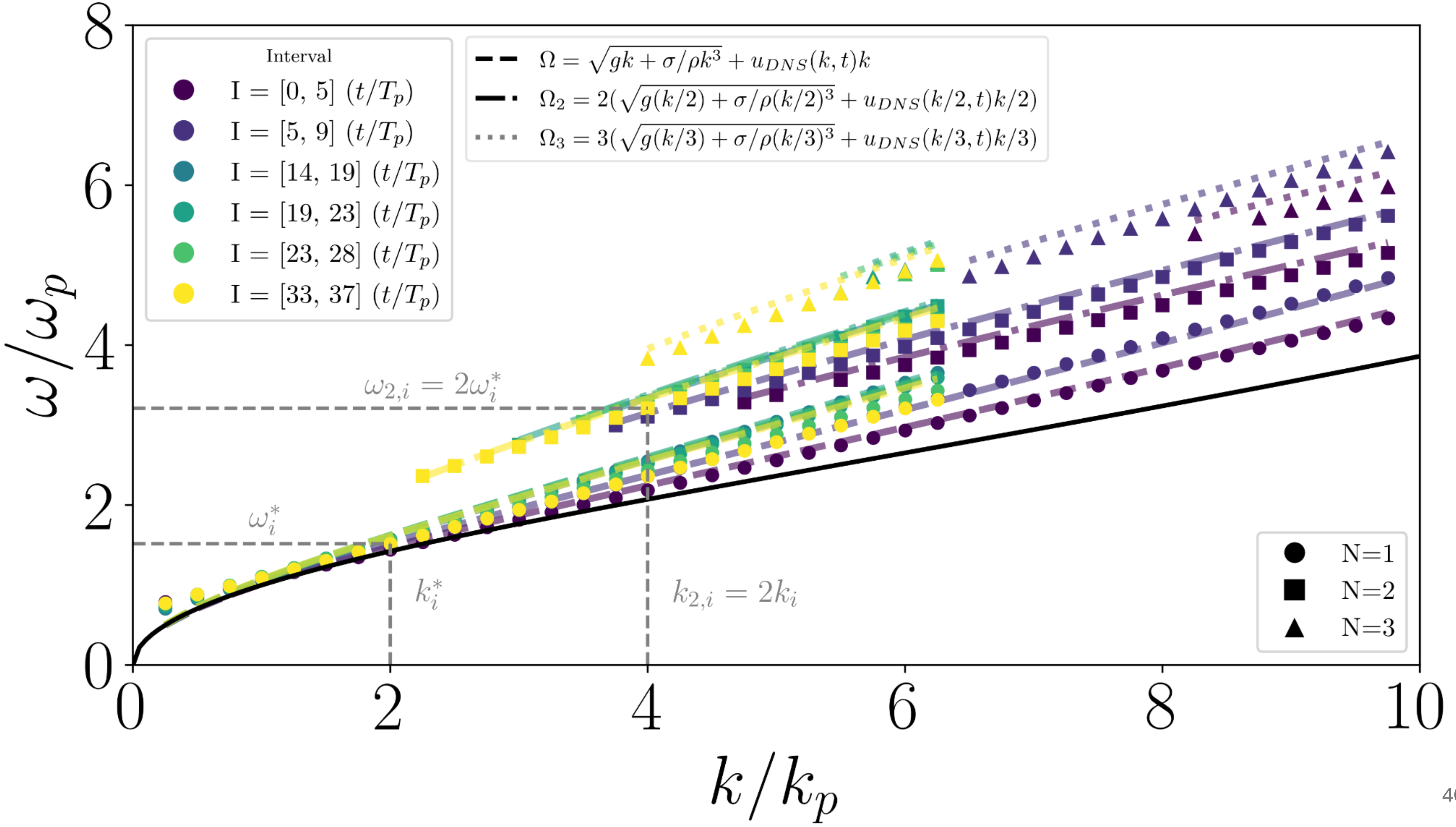
↓

Non-linear dispersion relation

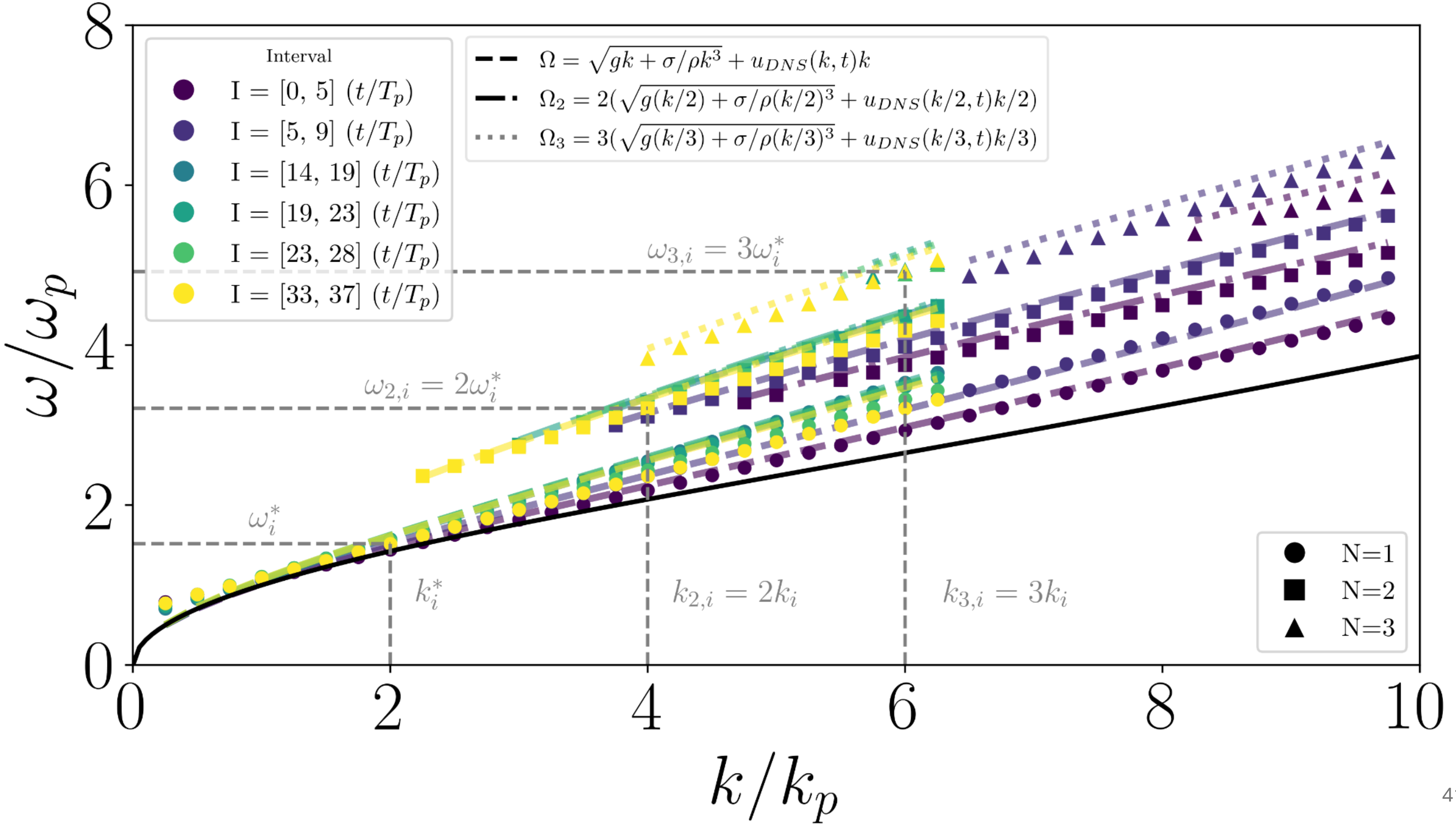
$$\Omega_N(k_N) = N\left[\sqrt{\frac{gk_N}{N} + \frac{\sigma}{\rho}\left(\frac{k_N}{N}\right)^3} + \frac{u_{eff}\left(\frac{k_N}{N}, t\right)k_N}{N}\right]$$

with the Doppler shift from Stewart & Joy (1974) $u_eff(k) = 2k\int_0^\infty u_L(z)e^{-2\text{kz}}dz$

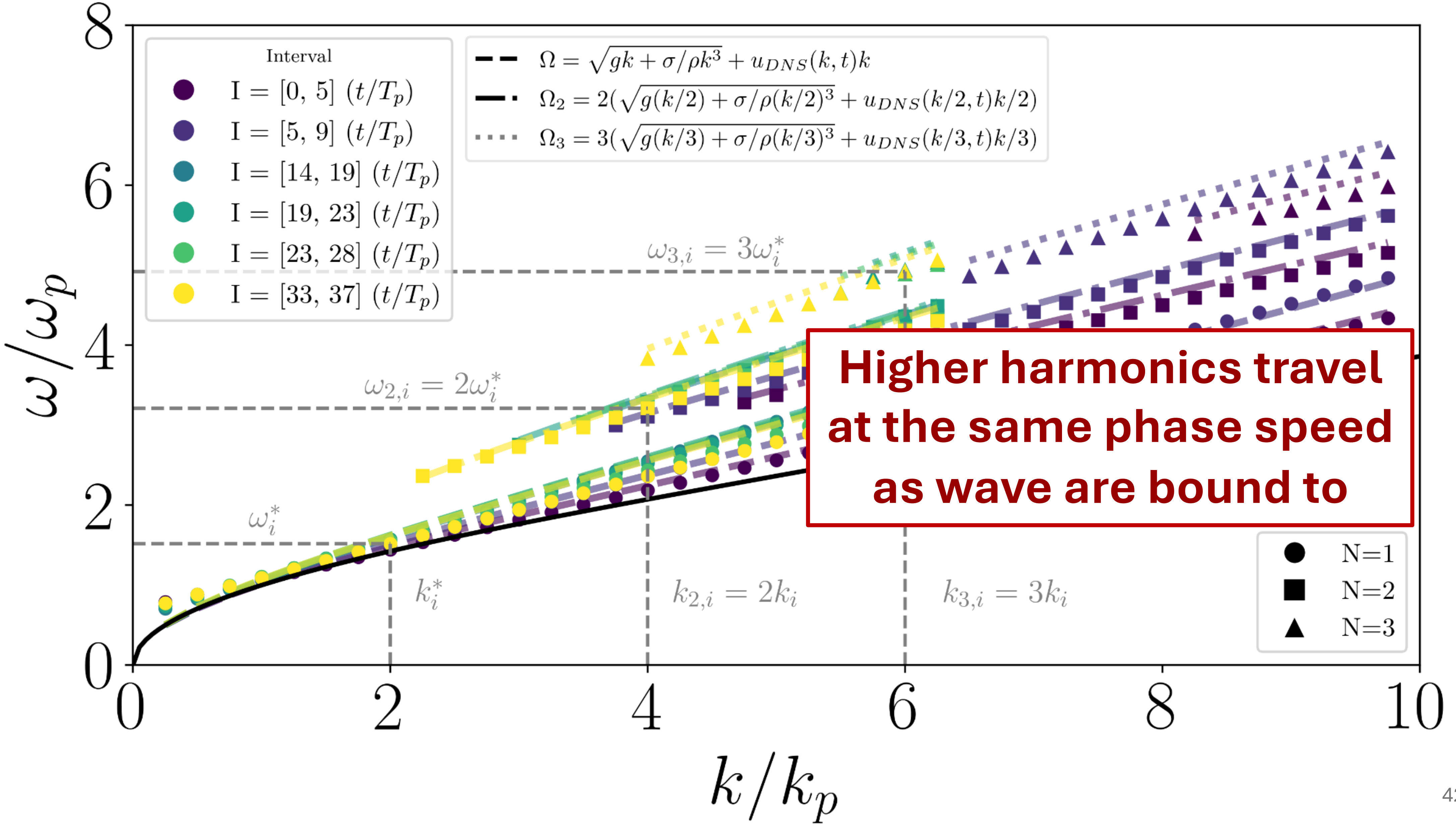
Phase speed of the higher bound modes



Phase speed of the higher bound modes



Phase speed of the higher bound modes



Stability and convergence

RMS surface elevation

$$\sigma_\eta = \sqrt{< \eta^2 >}$$

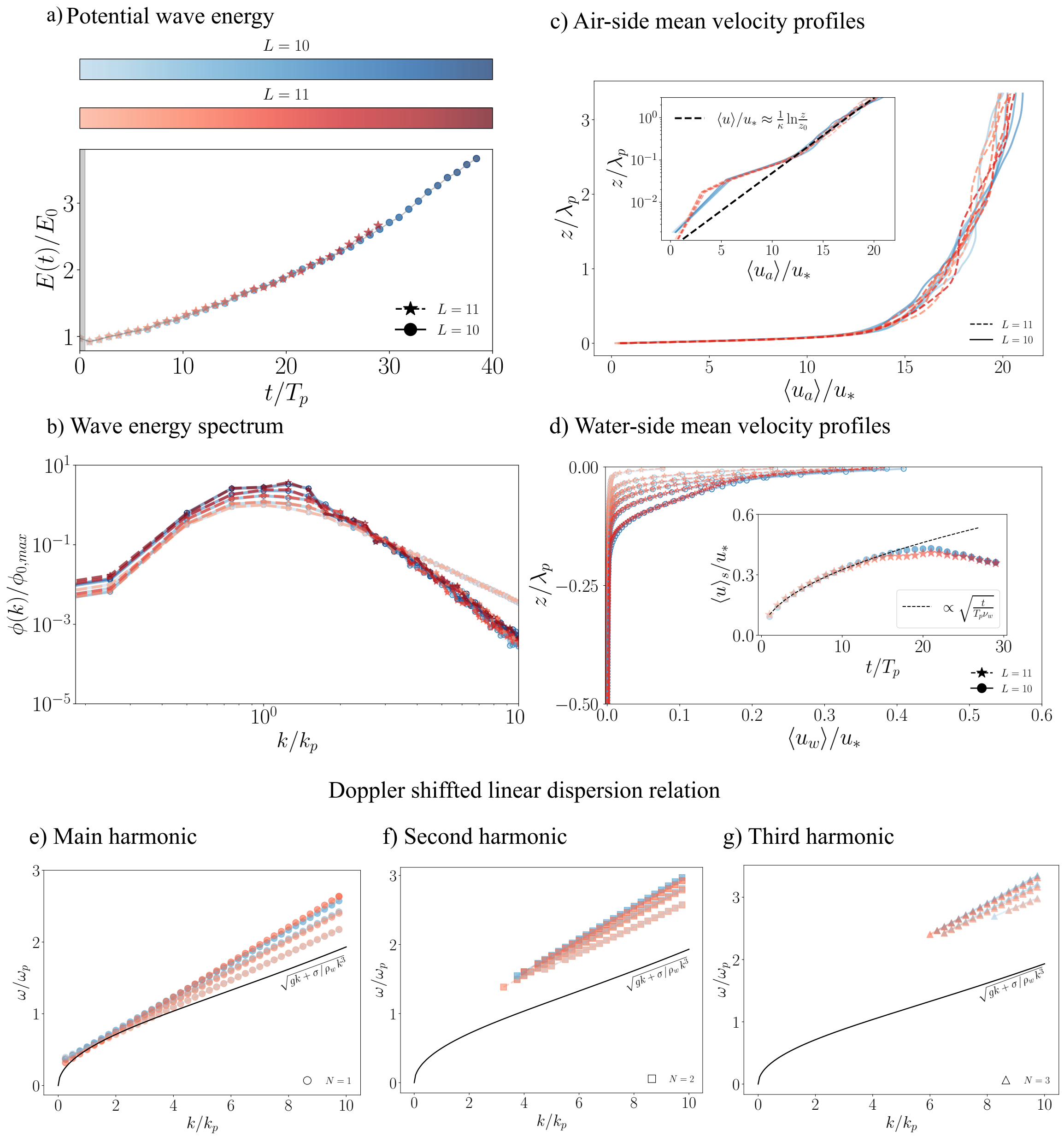
Viscous length

$$\delta_\nu = \lambda_p / \sqrt{Re}$$

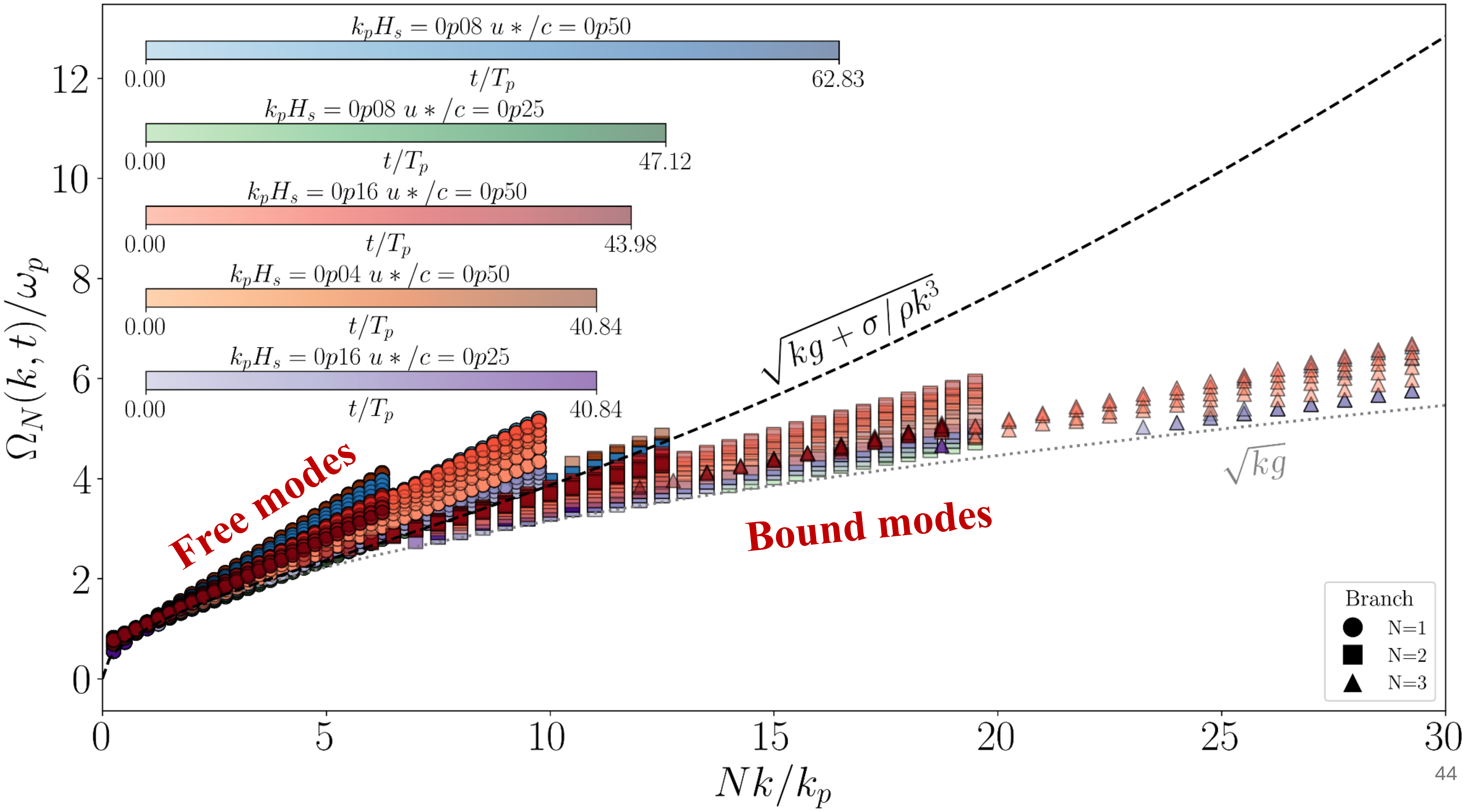
Kolmogorov length scale

$$\eta_k = \left(\frac{\nu_w^3}{\epsilon_{max}}\right)^{1/4} \qquad \epsilon = 2\nu_w S_{ij} S_{ij} \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

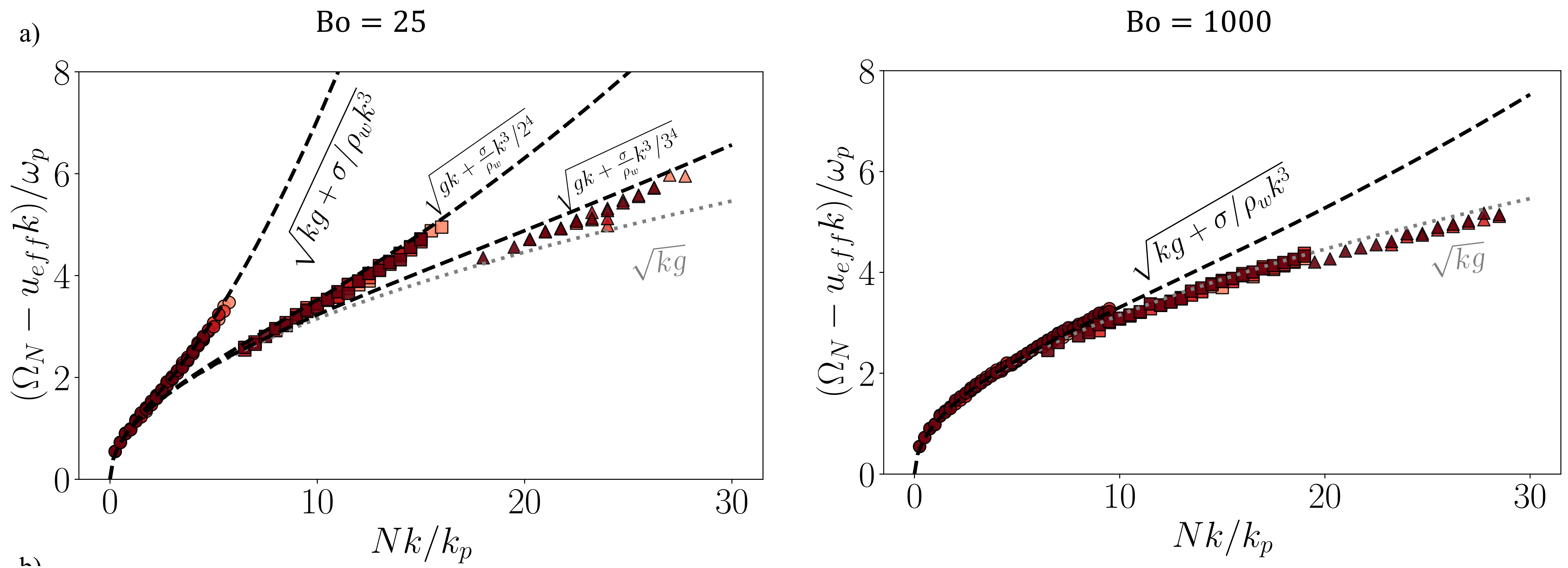
$k_p H_s$	u_*/c	Bo	L	σ_η/Δ	δ_ν/Δ	η_K/Δ
0.04-0.16	0.25-0.50	25, 200, 1000	10	1-2.3	1.6	2.2-2.6
0.16	0.50	200	11	1.5-4.2	3.2	2.2-2.7



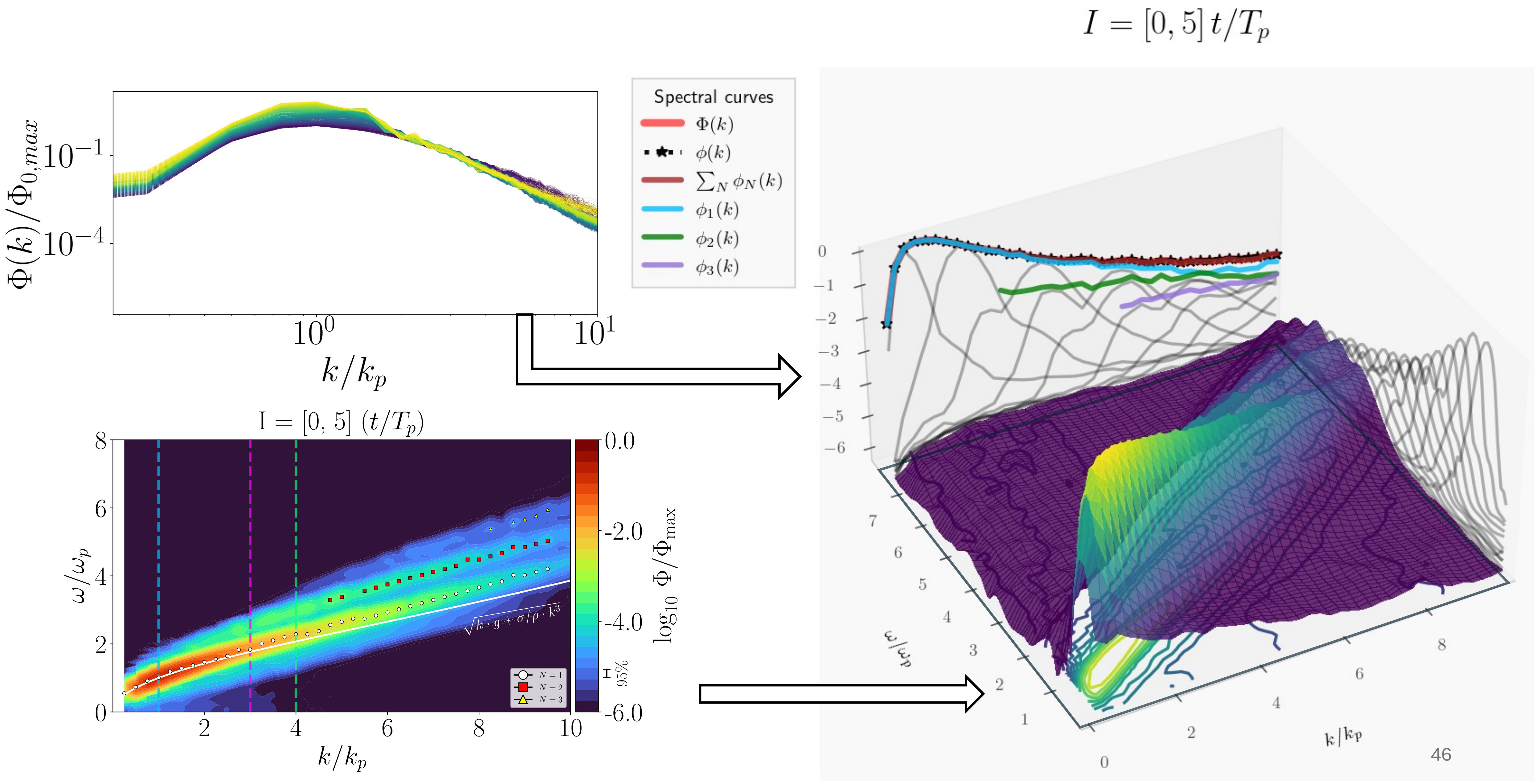
Does the non-linear dispersion relation hold across different wind and wave initial conditions?



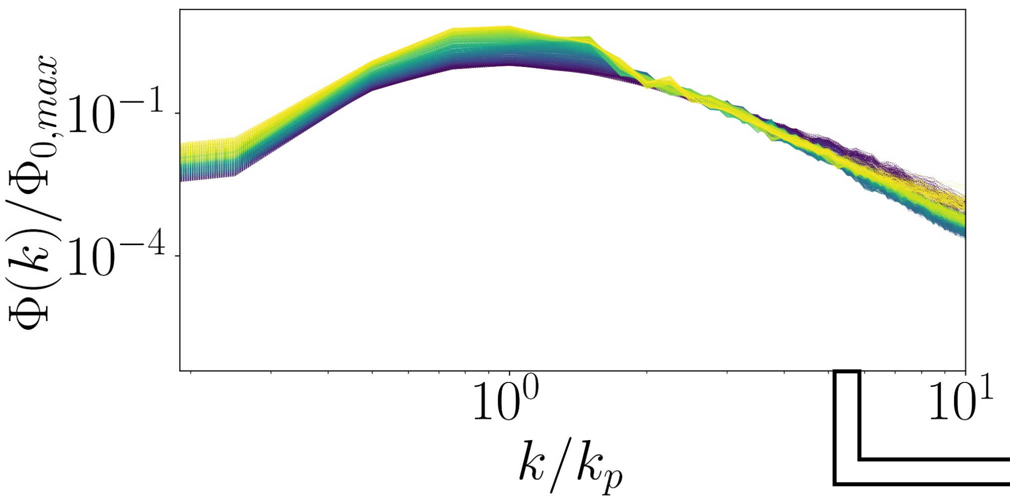
Different gravity-capillary regimes

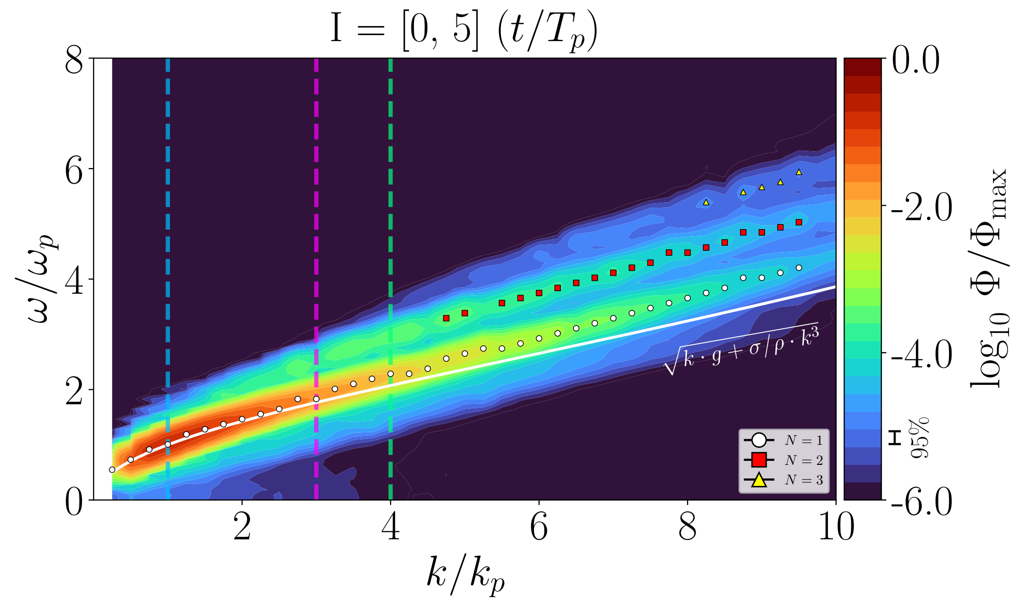


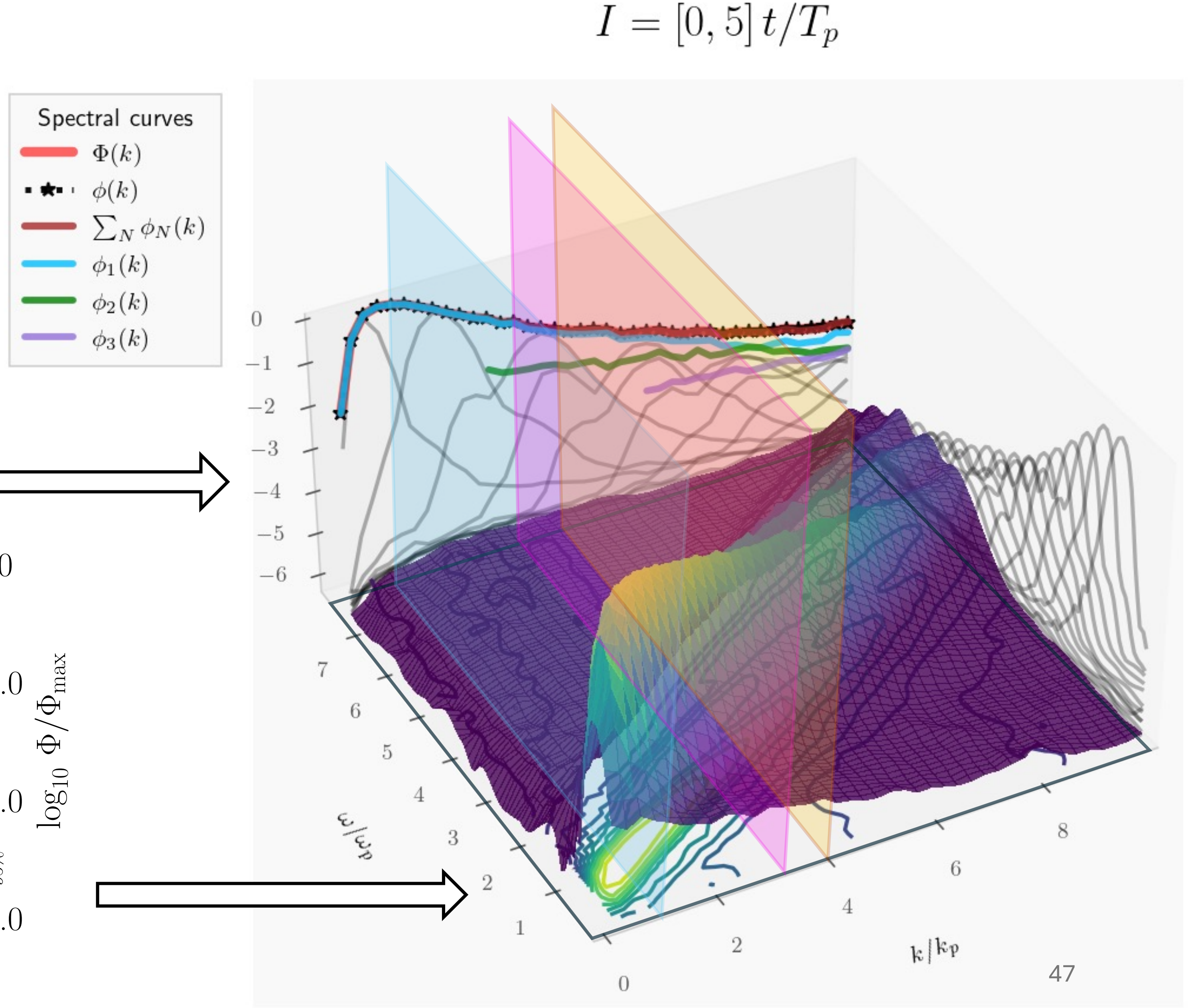
$(k-\omega)$ spectral analysis: growth of modes $\beta_\Phi(k)$ and $\beta_N(k)$



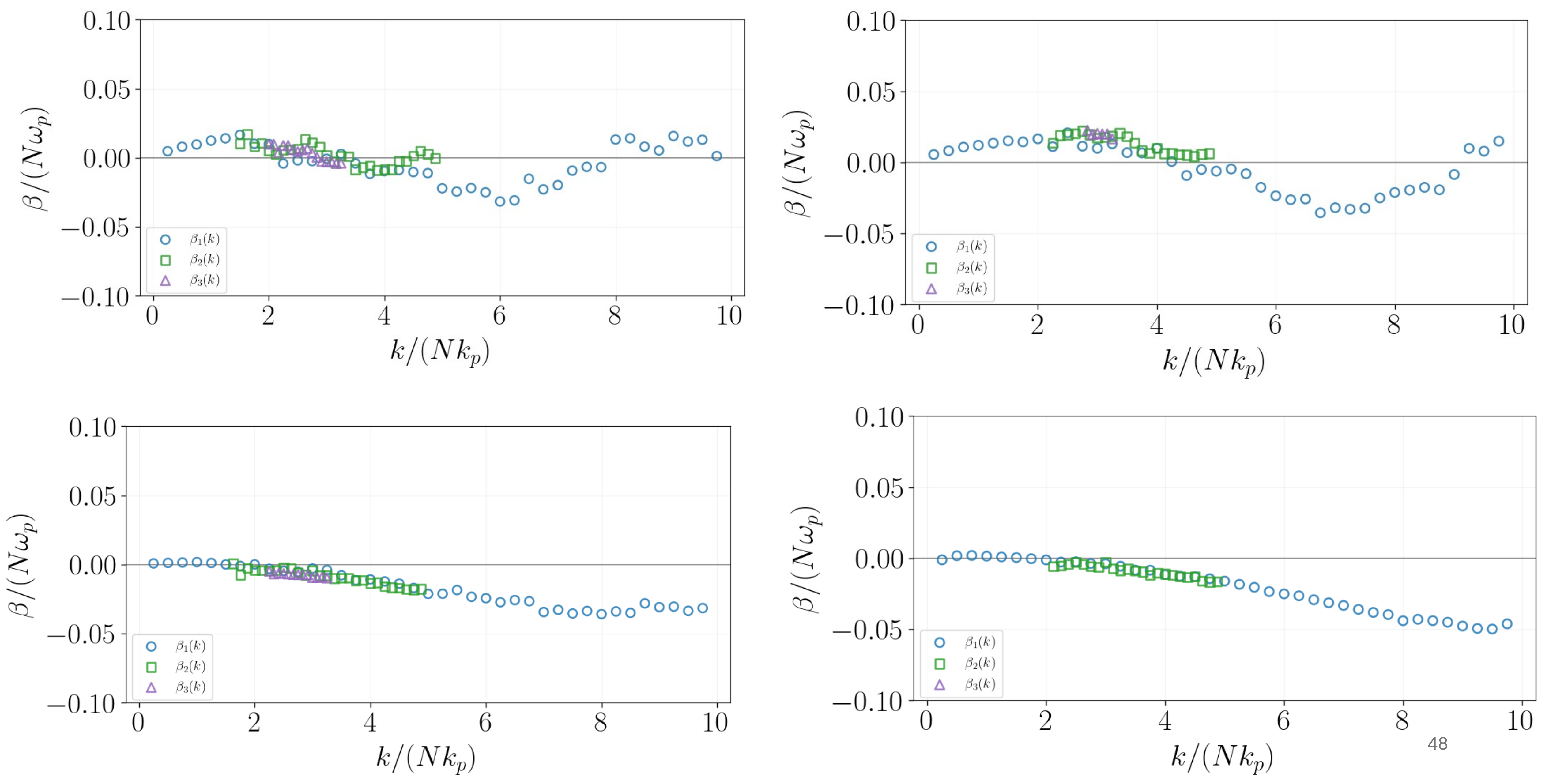
(k − ω) spectral analysis: growth of modes $\beta_{\Phi}(k)$ and $\beta_N(k, N)$







Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



Can we reconstruct $\beta(k)$ from $\beta_N(k, N)$ of each branch?

k space $\phi(k)$

Growth rate $\beta(k)$

$$\beta(k) = \frac{d}{dt}\ln\phi(k,t) = \frac{1}{\phi}\frac{\partial\phi}{\partial t}$$

$(k-\omega)$ space

$$\Phi(k,t) = \int \Phi(k,\omega',t)d\omega' \approx \underbrace{\sum_N \int_{B_N} \Phi d\omega' = \sum_N \Phi_N(k,t)}_{\text{branches}} \quad \text{where} \quad \Phi_N(k,t) = \int_{\omega\in B_N} \Phi(k,\omega',t)d\omega'$$

Growth rate $\beta_N(k)$

$$\beta_N(k) = \frac{d}{dt}\ln\Phi_N(k,t) = \frac{1}{\Phi_N}\frac{\partial\Phi_N}{\partial t} \longrightarrow \frac{\partial\Phi_N}{\partial t} = \beta_N\Phi_N$$

$$\Phi(k) = \sum_N \Phi_N(k) = \phi(k)$$

$$\longrightarrow \beta(k) = \frac{1}{\sum_N \Phi_N}\sum_N \frac{\partial\Phi_N}{\partial t}$$

$$\boxed{\beta(k) = \frac{\sum_N \beta_N\Phi_N}{\sum_N \Phi_N}}$$

Weighted average

Can we reconstruct β(k) from $\beta_N(k, N)$ of each branch?

k space $\phi(k)$

Growth rate $\beta(k)$

$$\beta(k) = \frac{d}{dt}\ln\phi(k,t) = \frac{1}{\phi}\frac{\partial\phi}{\partial t}$$

$$\Phi(k) = \sum_N \Phi_N(k) = \phi(k)$$

$$\beta(k) = \frac{1}{\sum_N \Phi_N}\sum_N \frac{\partial\Phi_N}{\partial t}$$

$(k-\omega)$ space

$$\Phi(k,t) = \int \Phi(k,\omega',t)d\omega' \approx \underbrace{\sum_N \int_{B_N}\Phi d\omega' = \sum_N \Phi_N(k,t)}_{\text{branches}}$$

where

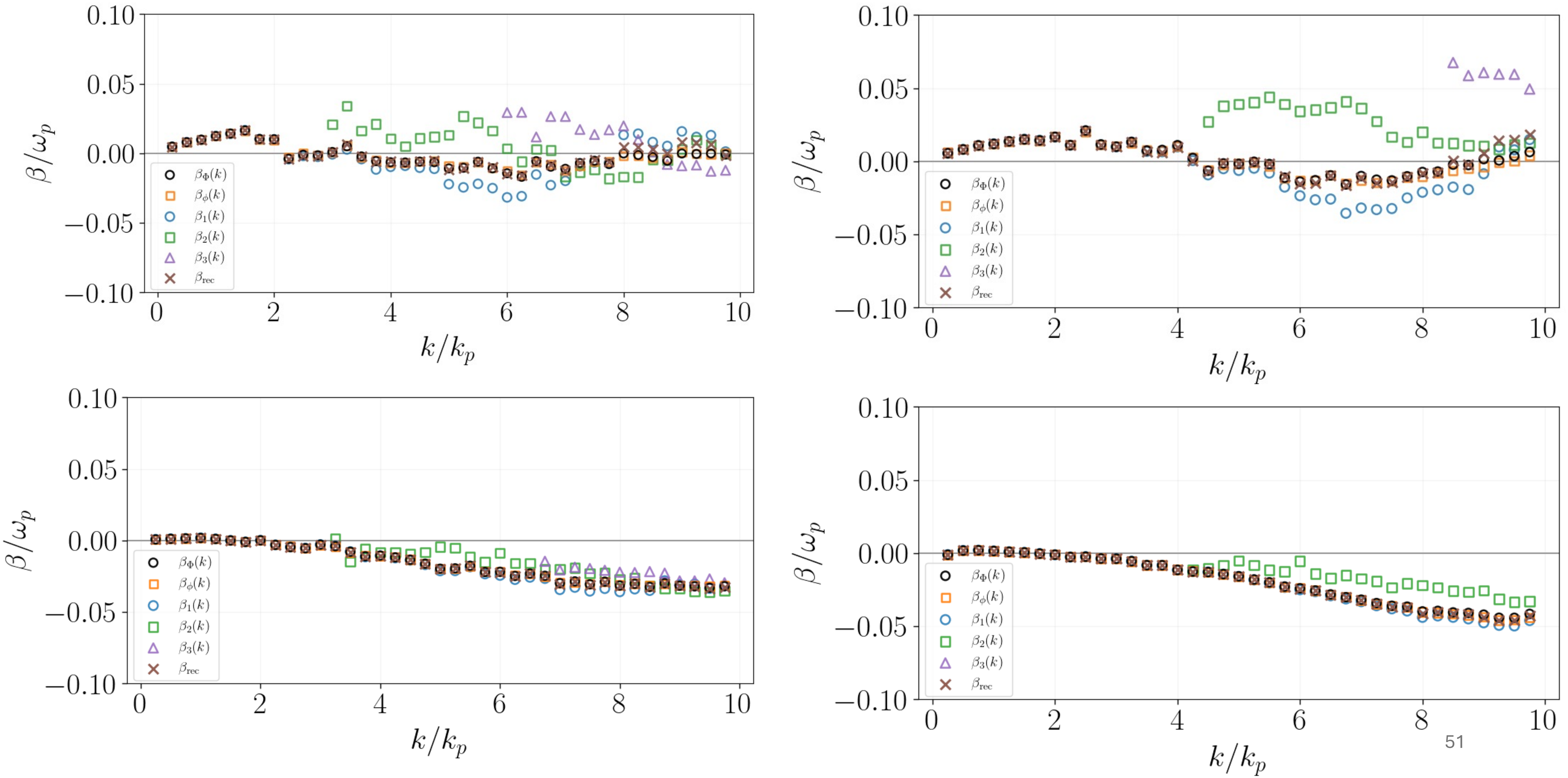
$$\Phi_N(k,t) = \int_{\omega\in B_N}\Phi(k,\omega',t)d\omega'$$

Growth rate $\beta_N(k)$

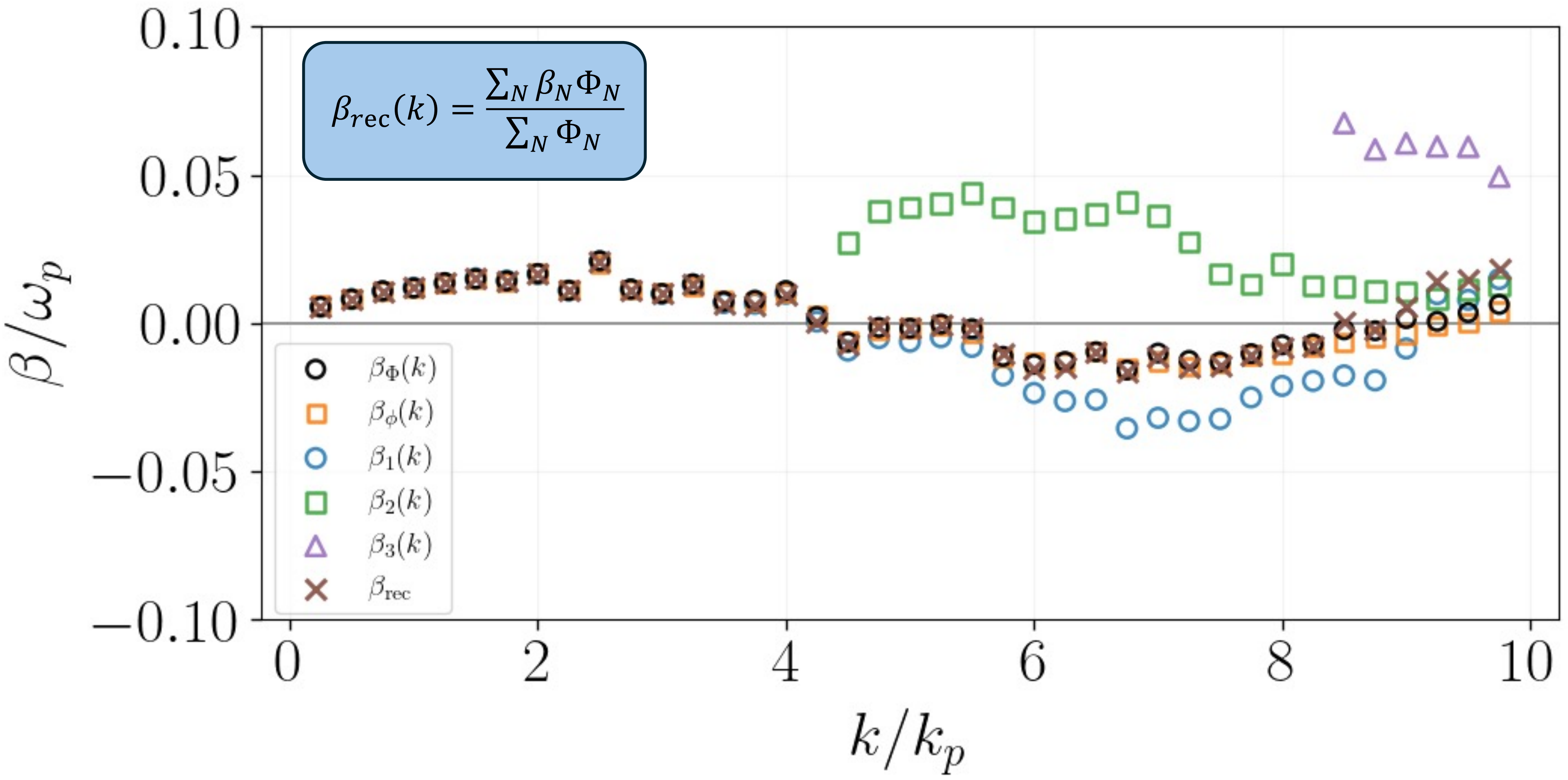
$$\beta_N(k) = \frac{d}{dt}\ln\Phi_N(k,t) = \frac{1}{\Phi_N}\frac{\partial\Phi_N}{\partial t} \longrightarrow \frac{\partial\Phi_N}{\partial t} = \beta_N\Phi_N$$

$$\beta_{rec}(k) = \frac{\sum_N \beta_N\Phi_N}{\sum_N \Phi_N}$$

Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



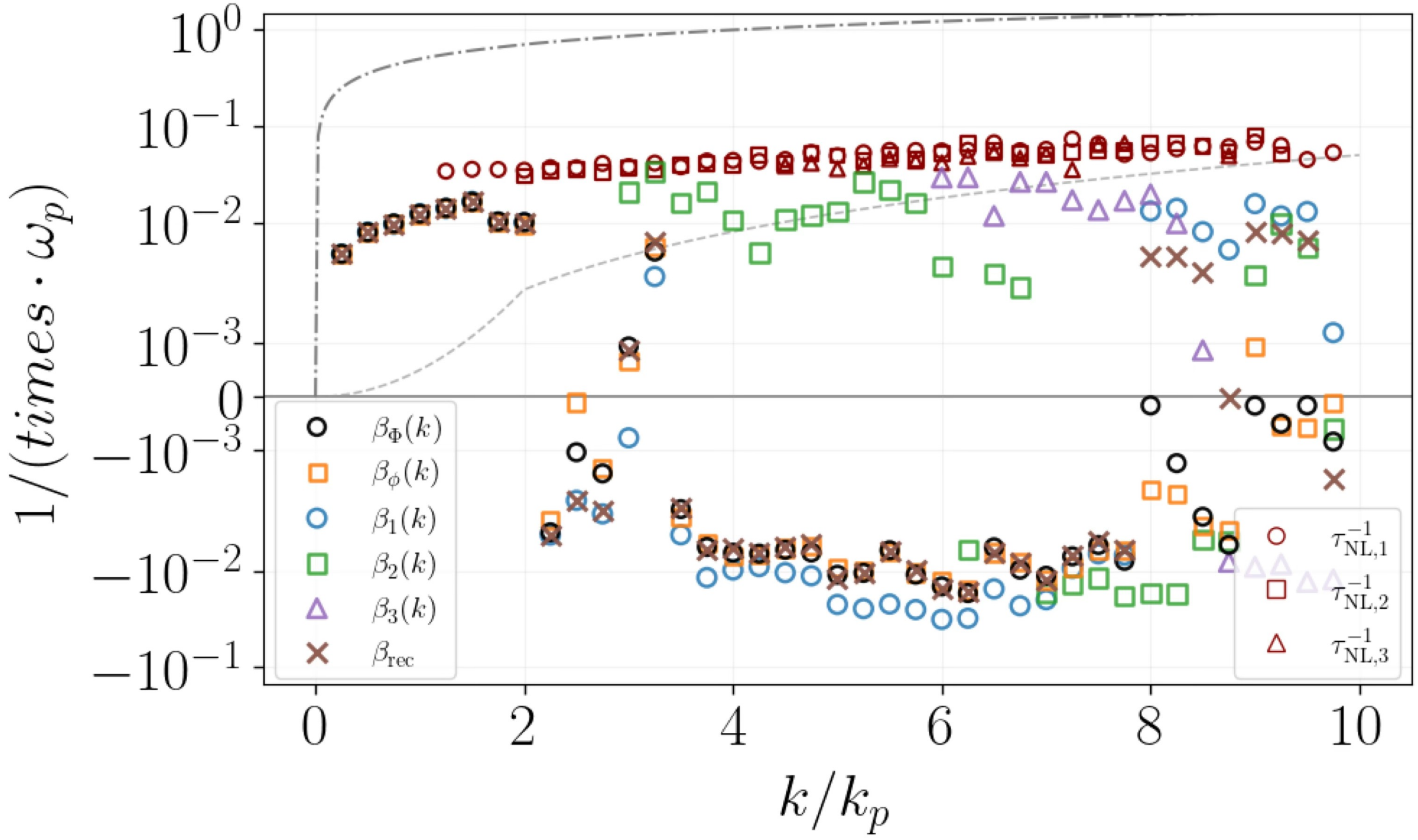
Energy Extraction: Growth rate per branch accounting for Doppler shift $\beta_N(k, N)$



Growth rate comparison $\beta_N(k, N)$

Wave growth rate $\beta_\phi(k), \beta_\Phi(k), \beta_{rec}, \beta_N(k)$

Inverse time scales $\tau_{NL,N}, \tau_{diss}, \tau_L$



Parserval theorem

$$\int \sum_N \Phi_N(k,t)dk = \int \Phi(k,t)dk = \int \phi(k,t)dk = var(\eta)$$