High Meadows Environmental Institute

MECHANICAL & AEROSPACE ENGINEERING

# Momentum and energy fluxes in wind-forced breaking waves

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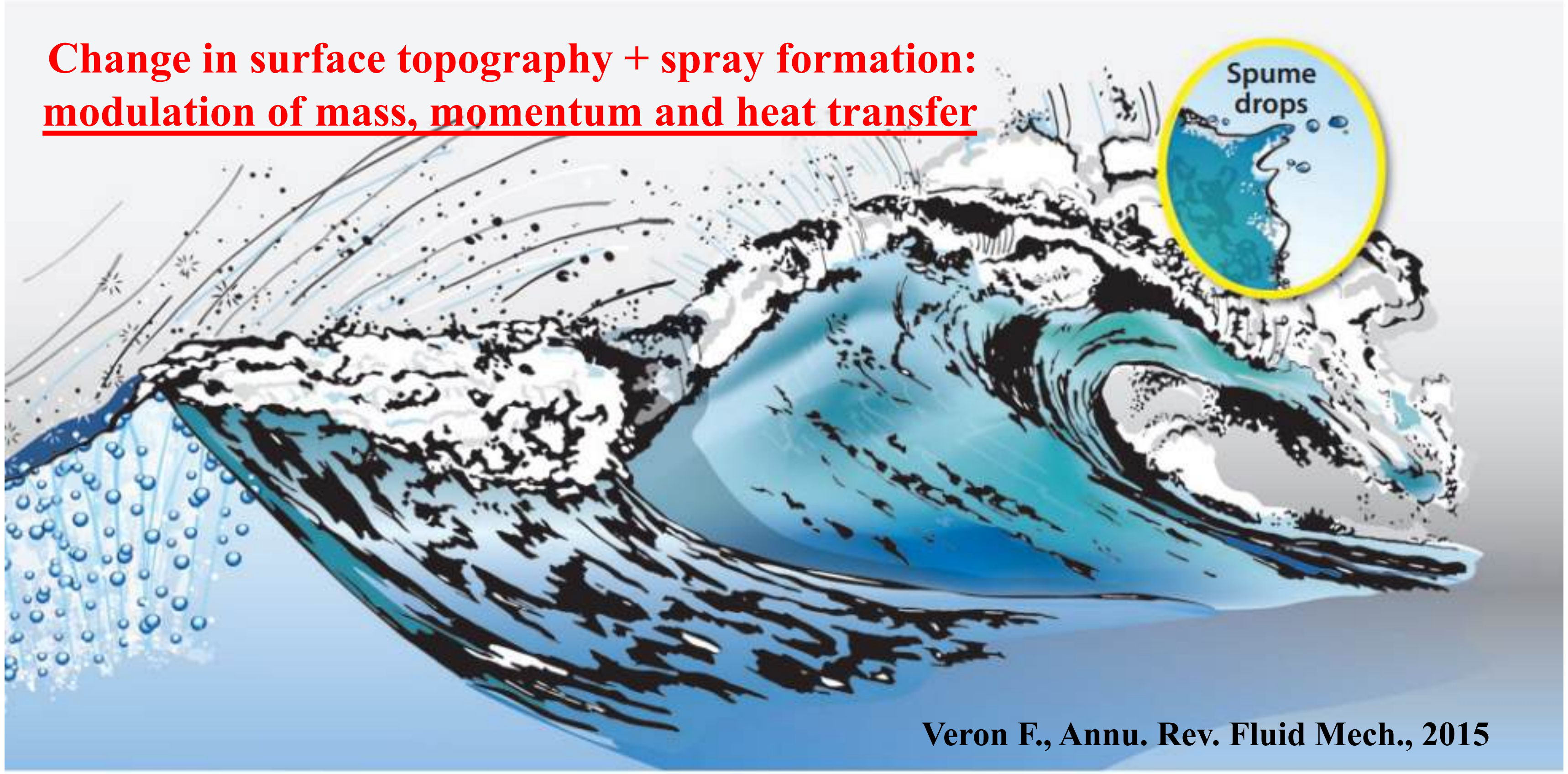
# Wind-forced breaking waves




- **Wind forcing** induces: *wave growth*, *steepening,* and eventually **wave breaking** with **energy transfer** to the water column.
- **Waves** and **wave breaking** modulate the exchanges of **momentum**, **energy** and mass at the ocean-atmosphere interface.

# Exchanged fluxes at high wind speeds

## Dimensionless momentum flux, $C_D$

$$C_D = \frac{\rho_a u_*^2}{\rho_a u^2(\bar{z} = 10\,m)}$$

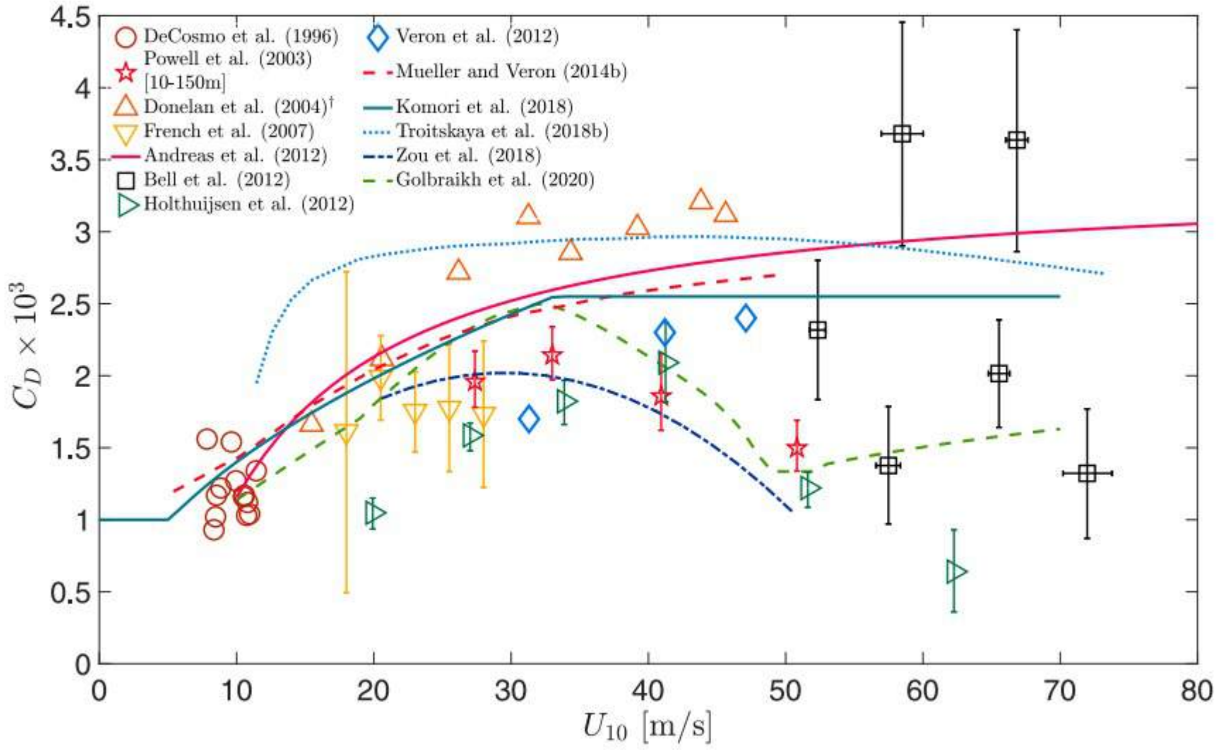
--> **Drag coefficient:** the imposed stress over **velocity $U_{10}$**.

**-->** **Uncertainty** in **(a)** the ***imposed stress*** and **(b)** in the ***velocity profile*** leads to a large data scattering.

## Energy transfer

--> **Modulation** of the **growth rate** by the wind forcing

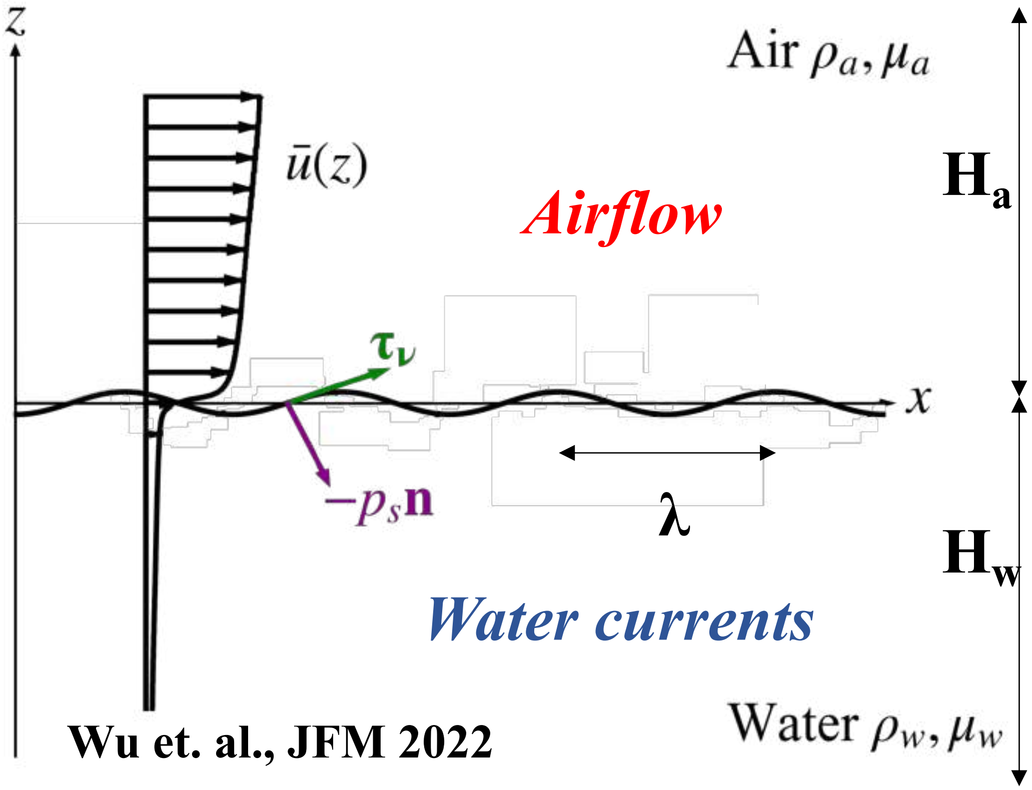
--> **Dissipation** induced by wave breaking




**"A Review of Parameterizations for Enthalpy and Momentum Fluxes from Sea Spray in Tropical Cyclones"** by S. Sroka and K. Emanuel

## Objectives: retrieve from first principles the energy and momentum fluxes

# Two-phase turbulent boundary layer (1/2)



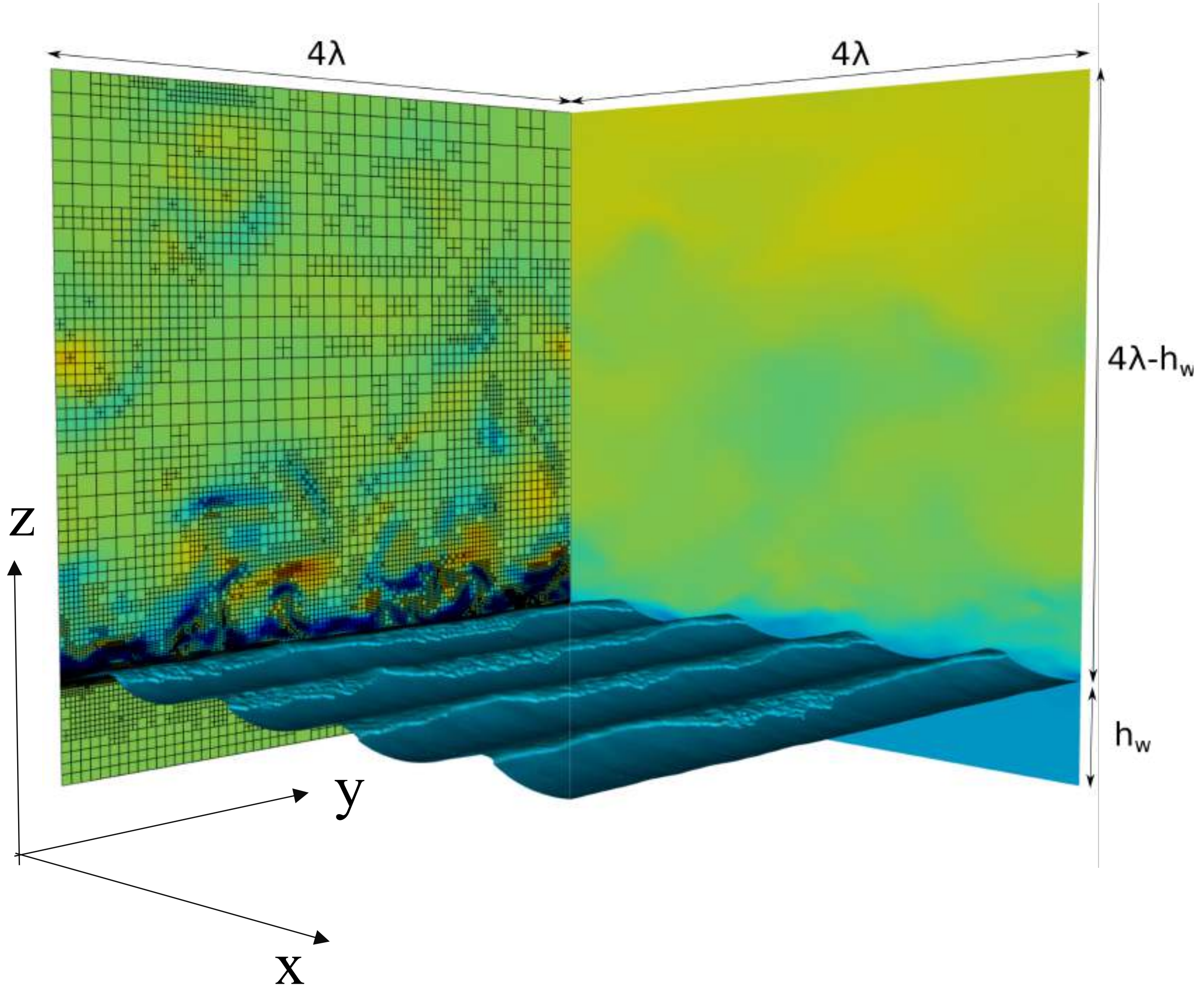

## Physical dimensionless parameters

- **Friction Reynolds number:** $Re_\tau = \frac{\rho_a u_* H_a}{\mu_a}$;
- **Wave Reynolds number:** $Re_{wave} = \frac{\rho_w c\lambda}{\mu_w}$;
- **Bond number:** $Bo = \frac{|\boldsymbol{g}|(\rho_w - \rho_a)\lambda^2}{4\pi^2\sigma}$;
- **Initial wave steepness:** $a_0 k$;
- **Friction velocity over wave speed:** $u_*/c$

- **Direct numerical simulations** of the two-phase system by solving the **Navier-Stokes equations**, i.e continuity and momentum equations with **Basilisk solver** (http://basilisk.fr/);
- **Fully-coupled simulations** without any model for the **wave shape**, i.e. waves can grow and break and **sub-grid model for turbulence**, i.e. all the scales are resolved.

# Two-phase turbulent boundary layer (2/2)

- **Initial condition in Air**: fully-developed turbulence (mean velocity independent of time);
- **Initial condition in Water**: potential flow solution of a third-order Stokes wave.




**Computational domain:**

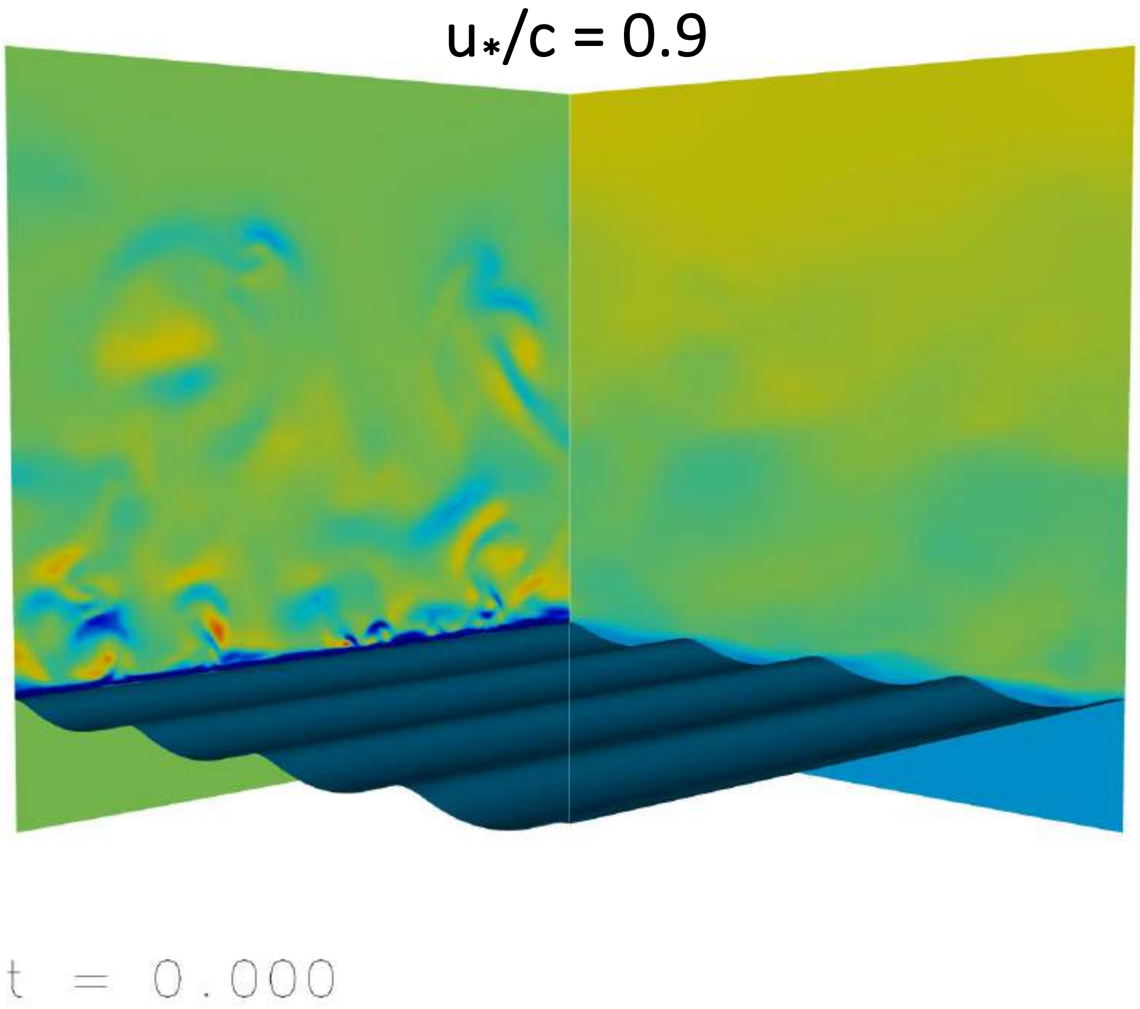
- $4\lambda \times 4\lambda \times 4\lambda, H_w \approx 0.64\lambda, H_a \approx 3.36\lambda$
- x-y: periodic directions; z: free-slip conditions;
- Grid resolution: $L^{10}$ (equivalent $1024^3$);

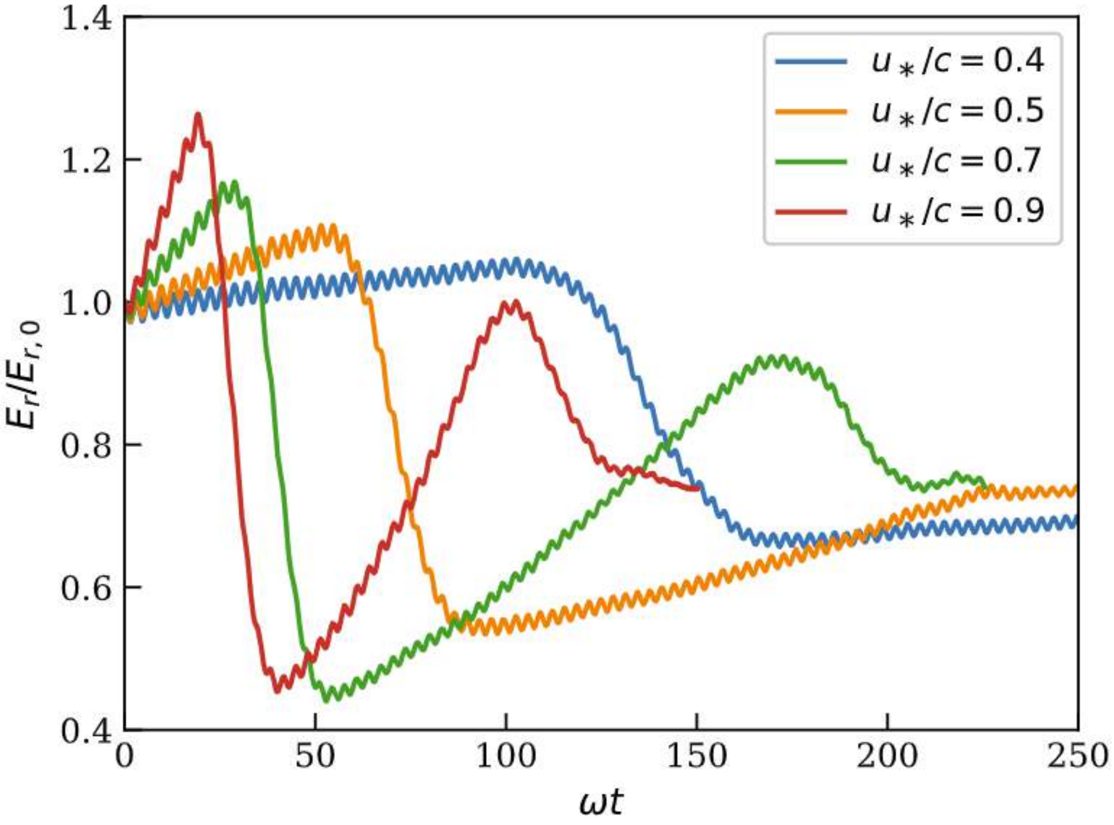
**We fix**:

$$Re_\tau = 720, Re_w = 2.5 \cdot 10^4, Bo = 200, a_0 k = 0.3$$

**We vary** (in the **high-wind speed** regime):

$$\frac{u_*}{c} = 0.3 - 0.4 - 0.5 - 0.7 - 0.9;$$




$$E_w(t) = \frac{1}{2}\rho_w|\boldsymbol{g}|a^2(t) \qquad a^2(t) = \frac{2}{A}\int(\eta - \bar{\eta})^2 dA$$

**<u>The instantaneous change in $E_w(t)$, i. e. change in a(t)k</u>**

(1) Affects the exchanged **momentum**, i.e. ***pressure and viscous forces***, between air and water;

(2) Modulates the **airflow**;

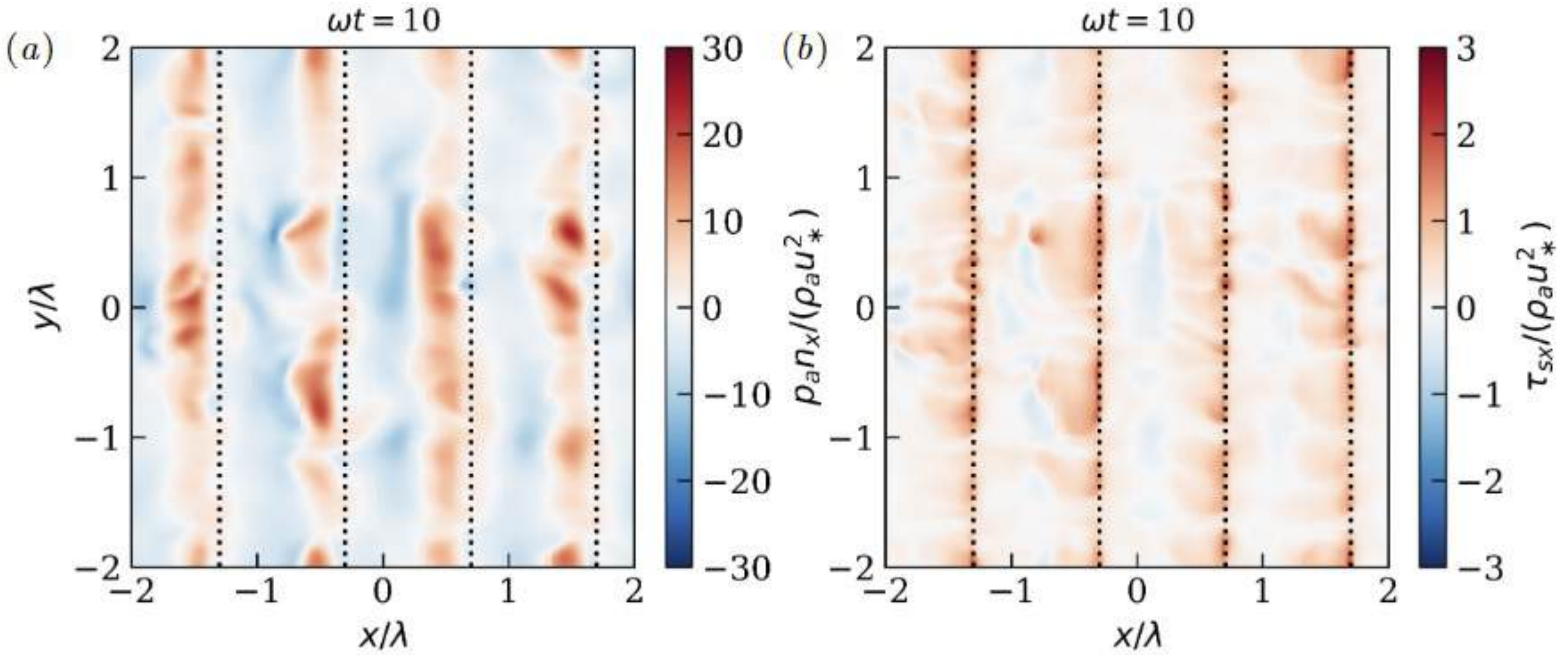
(3) Affect the **energy transfer** to the water column.

# (1) Momentum fluxes/exchanged forces

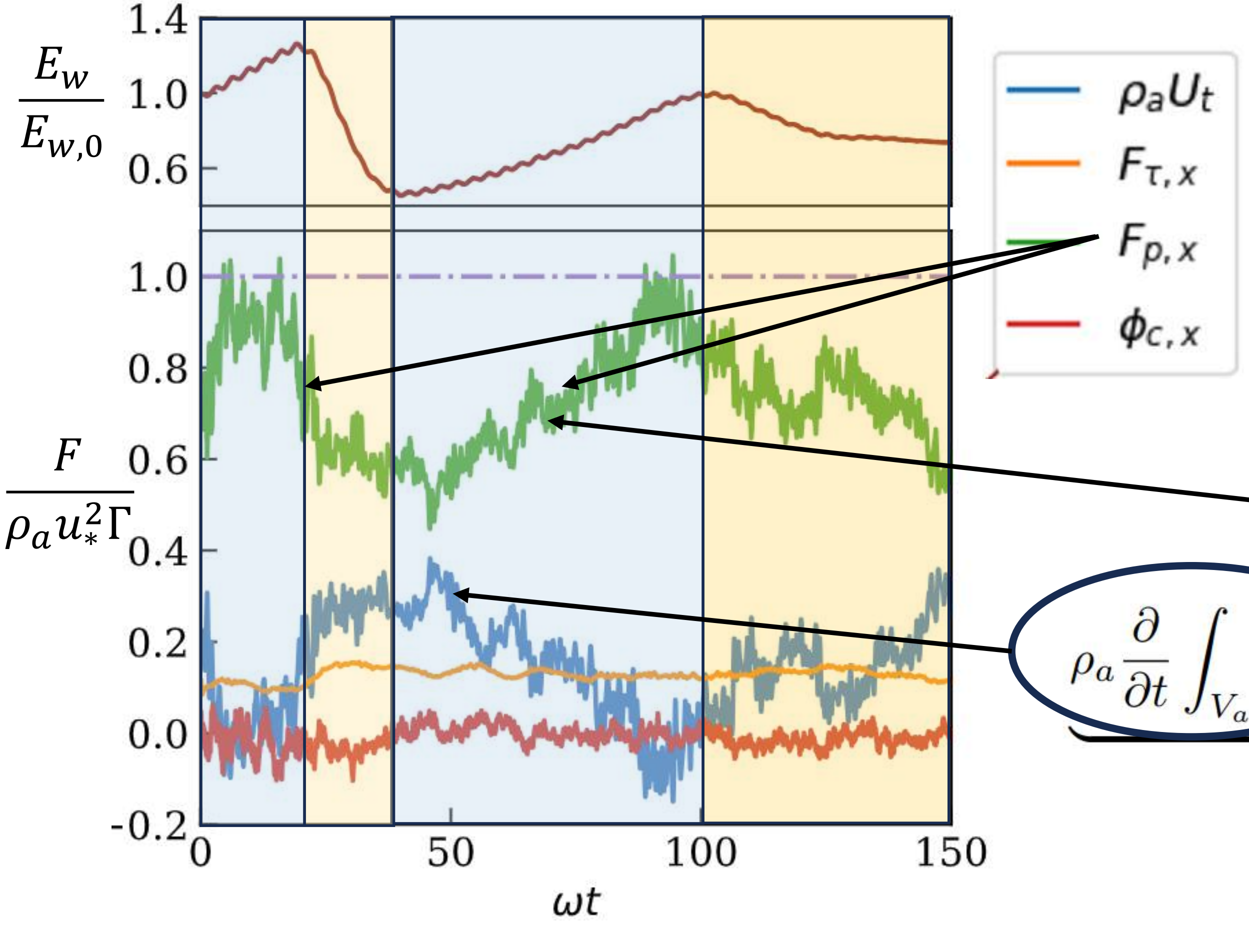
We decompose the streamwise force in a **pressure** and **viscous contribution** ($\boldsymbol{\tau} = \mu_a(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathbf{T}})$)

$$F_x = \underbrace{-\int_\Gamma p\boldsymbol{n}\cdot\boldsymbol{e}_x\,dA}_{F_{p,x}\,=\,\textbf{Pressure contribution}} + \underbrace{\int_\Gamma (\boldsymbol{\tau}\boldsymbol{n})\cdot\boldsymbol{e}_x dA}_{F_{\tau,x}\,=\,\textbf{Viscous contribution}}$$

$\boldsymbol{n} = (n_x, n_y, n_z)$: normal vector at the interface $\Gamma$, $\boldsymbol{e}_x = (1, 0, 0)$



# (1a) Momentum fluxes/exchanged forces




**Growing stage:** increase in the pressure force

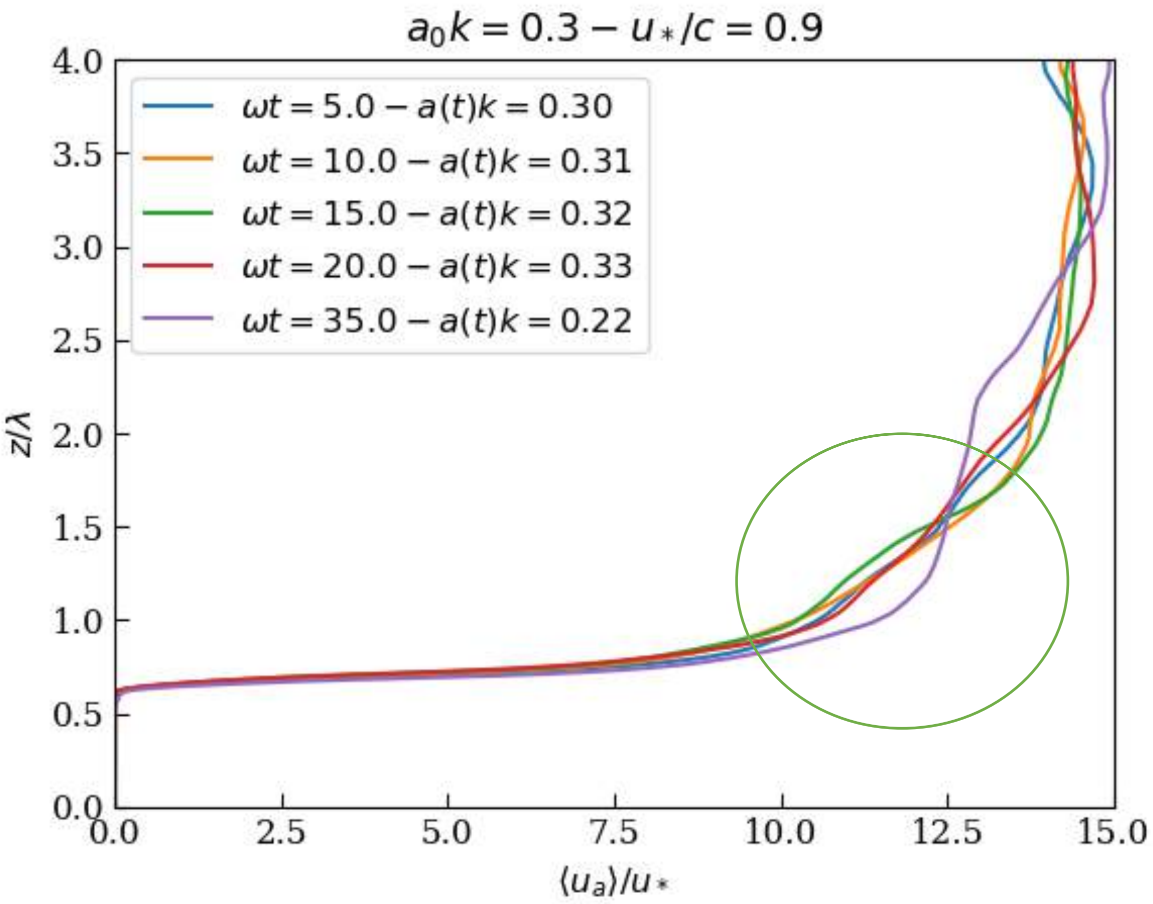
**Breaking stage:** drop in the pressure force

***Streamwise momentum budget in the air***

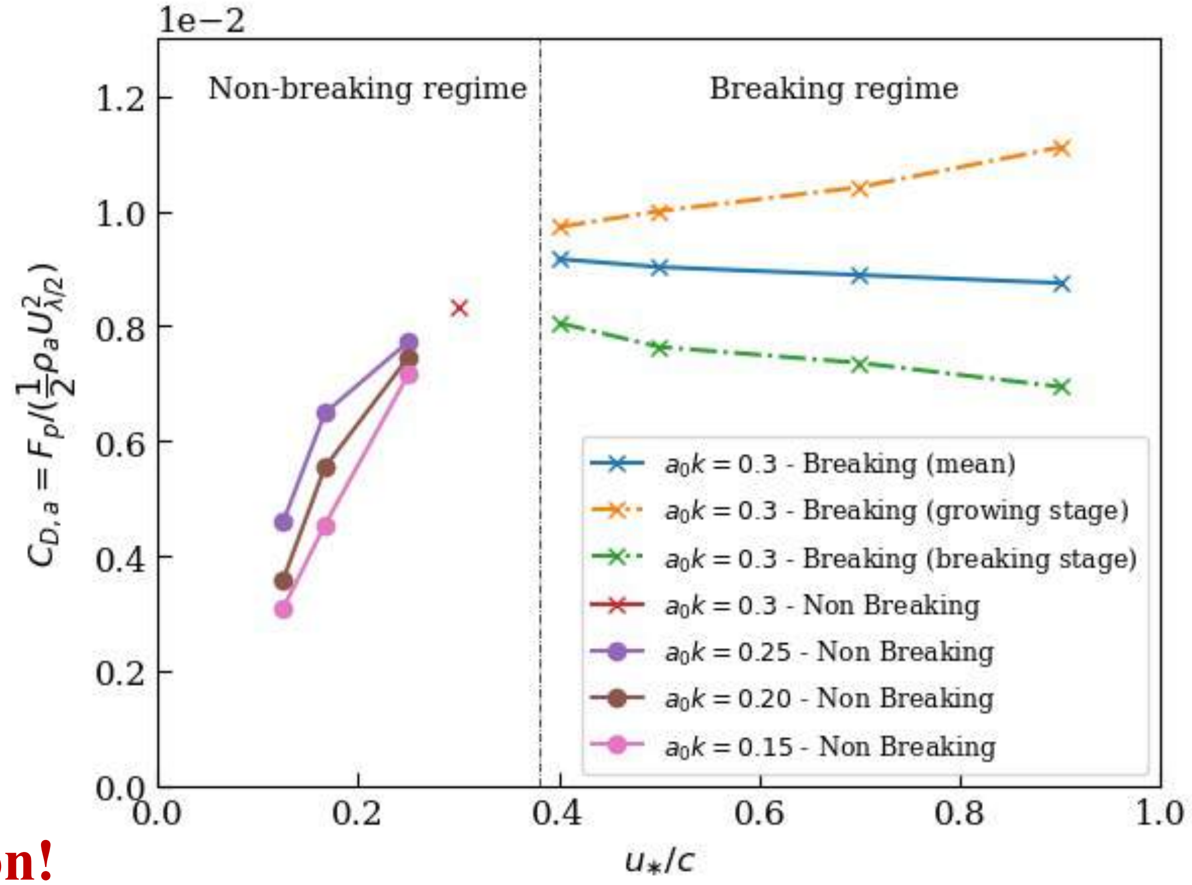
$$\underbrace{\rho_a \frac{\partial}{\partial t}\int_{V_a} u dV - \rho_a u_*^2 \frac{V_a}{4\lambda - h_w}}_{\rho_a U_t} + \underbrace{\rho_a \int_{\Gamma} \nabla \cdot (u\mathbf{u}_{ri}) dS}_{\phi_{c,x}} = F_{p,x} + F_{\tau,x}$$

the **compensation for pressure force** comes from the change in the **mean flow** and partially from the **viscous contribution** → **Airflow modulation**

# (2) Airflow modulation and drag coefficient (1/2)




$$C_{D,a} = \frac{\bar{F}_p}{\frac{1}{2}\rho_a \Gamma \bar{U}^2\ (z = \frac{\lambda}{2})}$$



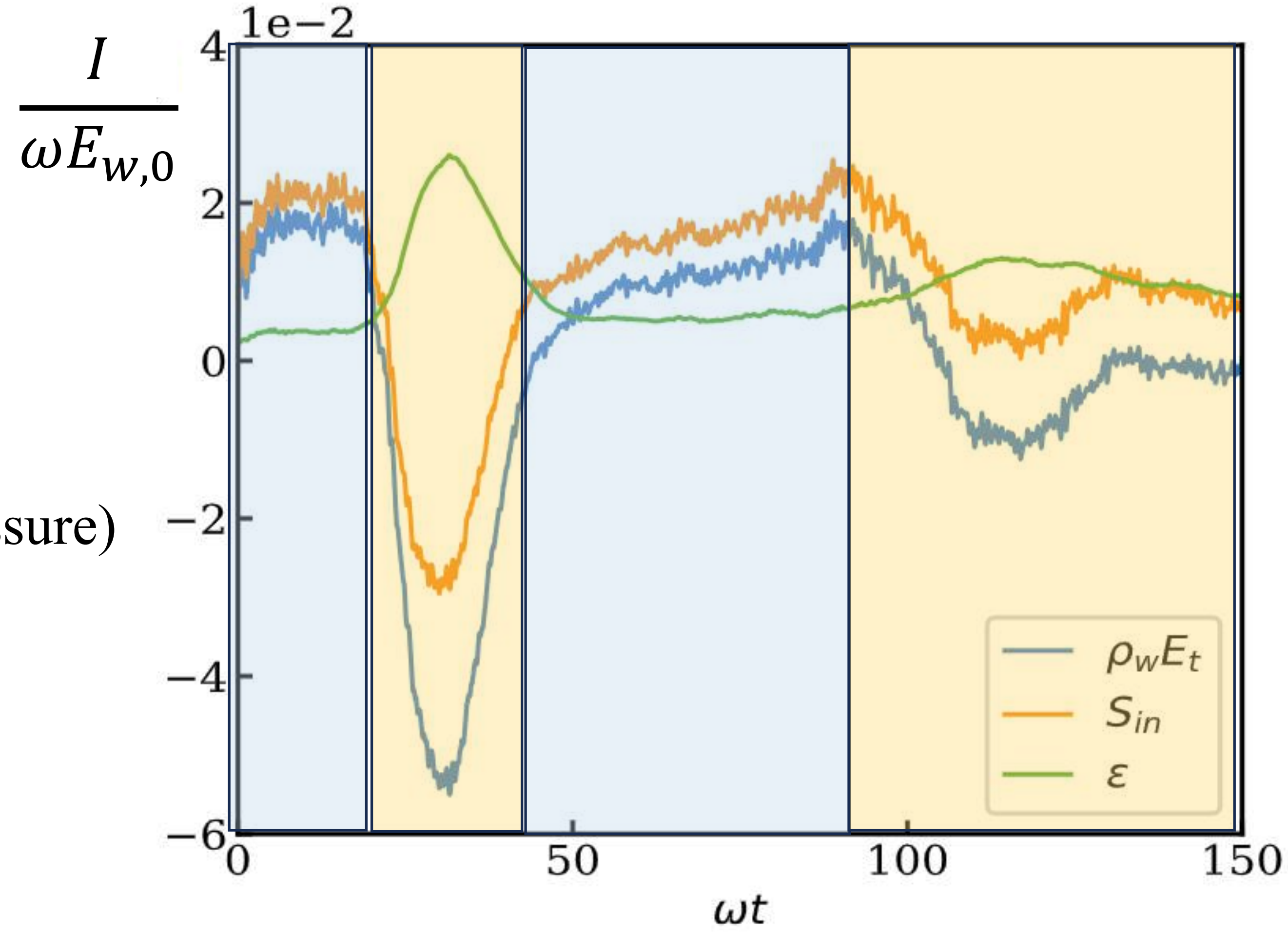

**The flow locally accelerates at the breaking.**

**Wave breaking is the driver behind $C_{D,a}$ saturation!**

# (3) Energy transfer to the water column

$$\frac{\partial e_w}{\partial t} = S_{in} - \varepsilon$$

- $e_w = \frac{1}{2}\rho_w|\boldsymbol{g}|a^2(t)$: wave energy
- $S_{in} = S_\tau + S_p$: input (viscous stress and pressure)
- $\varepsilon$: Dissipation --> $\varepsilon = \mu_w\left(\partial_i u_j + \partial_j u_i\right)^2$




- During the **growing stage**, the energy input $S_{in}$ is positive
- During the **breaking stage**, the energy input $S_{in}$ not only is much smaller than the dissipation but it turns to become negative

# (3a) Wind inputs

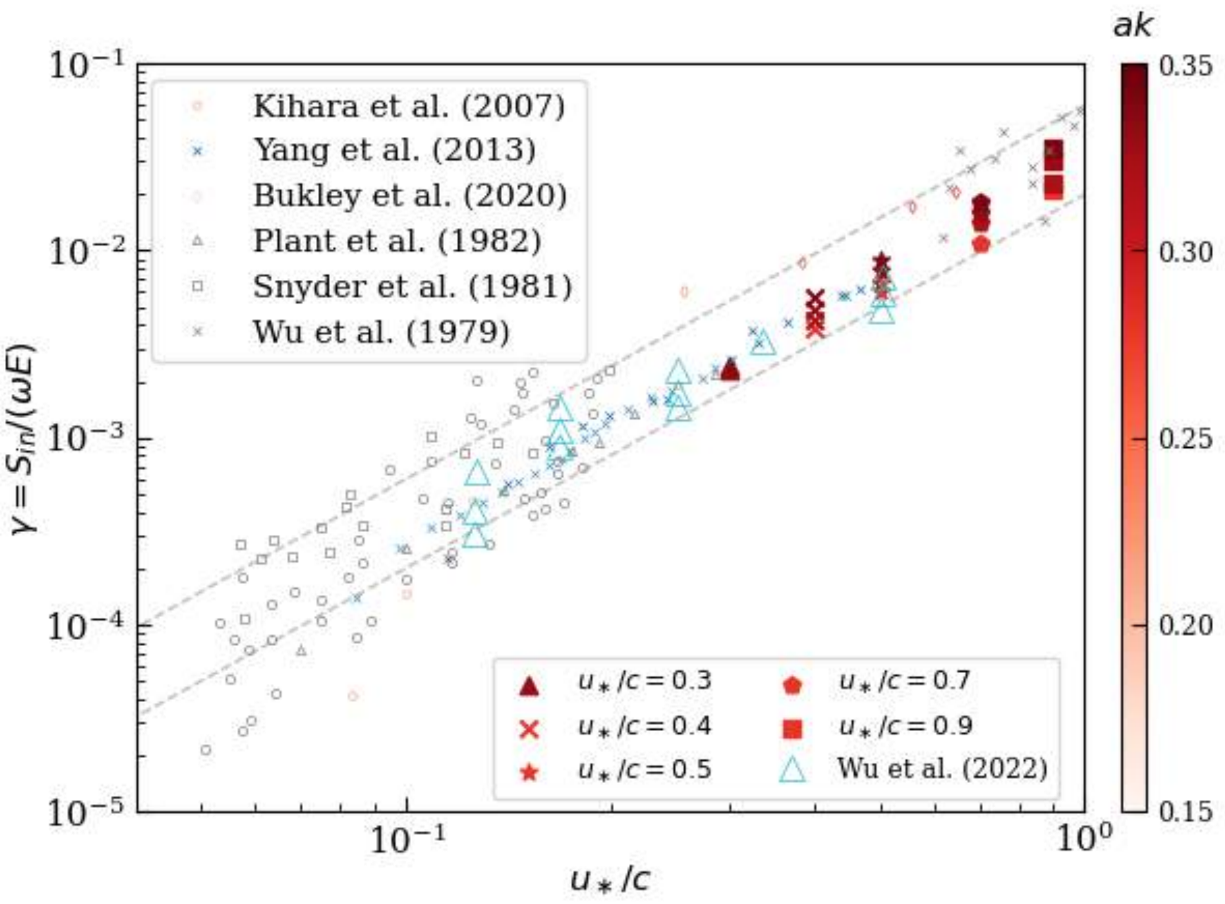
- From the total energy input, we can estimate the grow rate as

$$\gamma = \frac{S_{in}}{\omega E} = \frac{2S_{in}}{\omega g \rho_w a^2}$$

- **Miles scaling**: we can approximate $S_{in} \approx cF_p$, we can derive an expression for $\gamma$

$$\gamma = \frac{S_{in}}{\omega E} = \beta \frac{u_*^2}{c^2}\frac{\rho_a}{\rho_w}$$

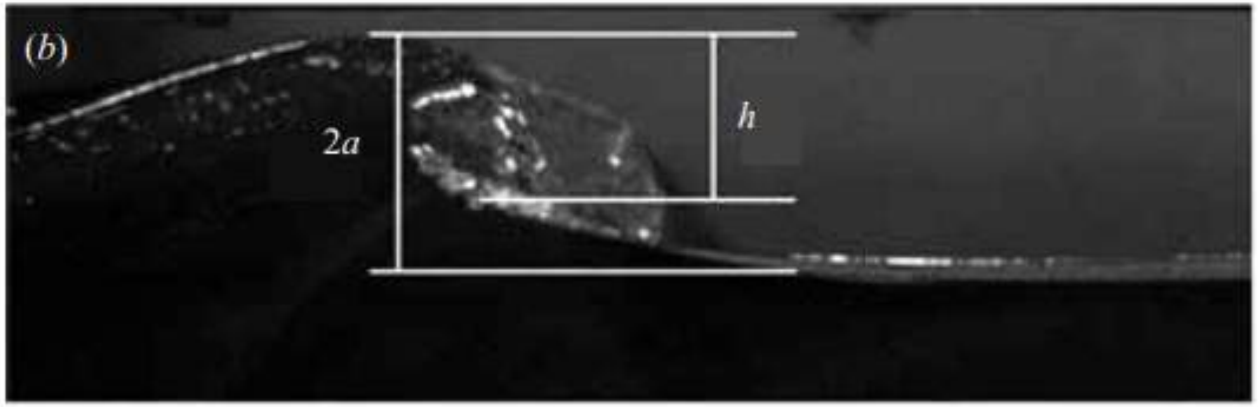
$\beta = 16 - 50$




Good agreement between the **DNS** and theoretical **Miles scaling** for different $u_*/c$ and $ak$

# (3b) Energy dissipation (1/3)

- **Dissipation** is required for **parametrization in operational wave models**.




- $\varepsilon = \chi_0 w_c^3 / l_c$ with $w_c = \sqrt{2 g l_c}, l_c = h$ with $h$ the breaking height, $w_c$ is the **ballistic velocity** (Melville, JFM 1994 and Drazen, JFM 2008).

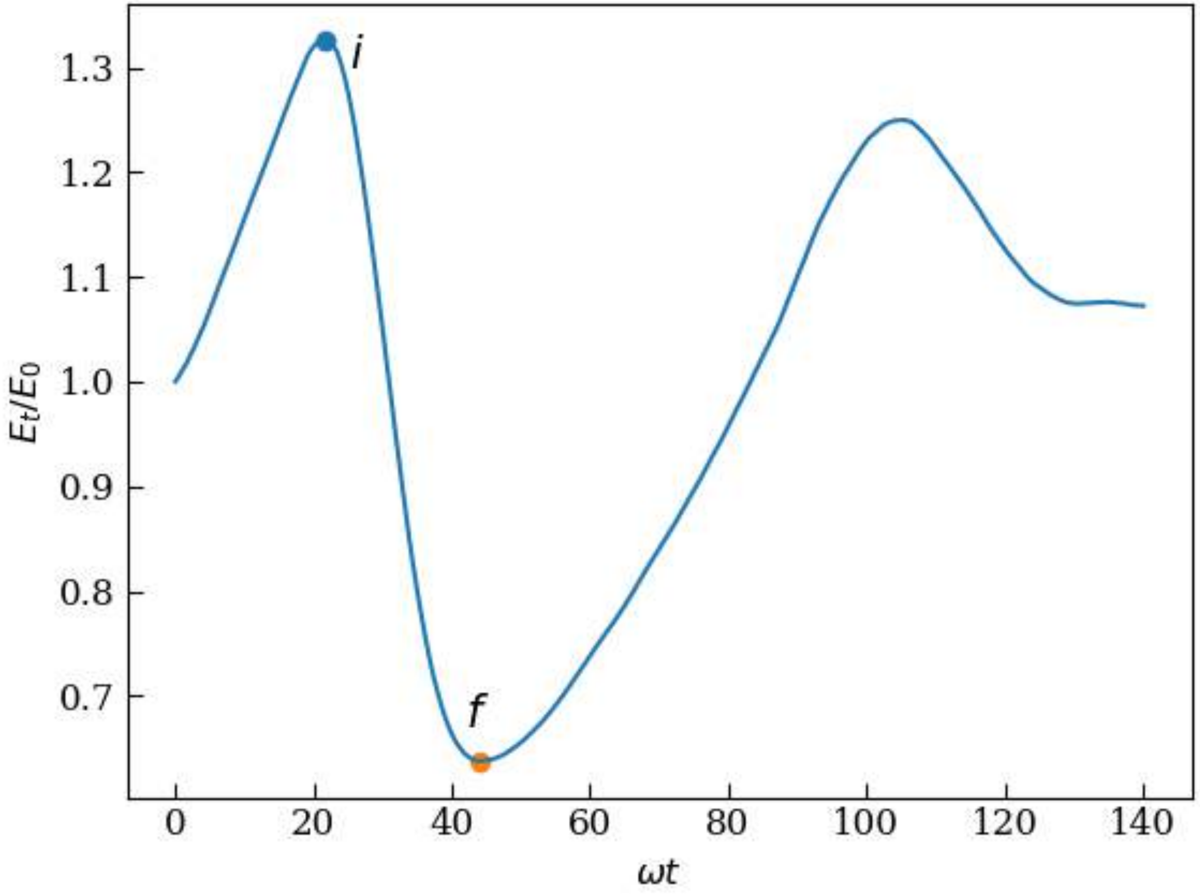
- In JFM-2008, Drazen suggested $\left(\frac{\varepsilon}{L_0}\right) = \varepsilon_l = \rho_w A \varepsilon$ with $A = \pi h^2/4$ (cylindrical cloud turbulence)

$$\varepsilon_l = \frac{b \rho_w c^5}{g}$$

- **Breaking parameters:** $b = \chi_1 (S - S_0)^{2.5}$ with $S = (hk), \chi_1 = O(1)$ and $S_0 = 0.08$.

# (3b) Energy dissipation (2/3)

This correlation was developed for **breaking waves**. Is it valid for **wind-forced breaking waves**?




$$\varepsilon_l L_0 = (E_i - E_f)/(t_i - t_f)$$

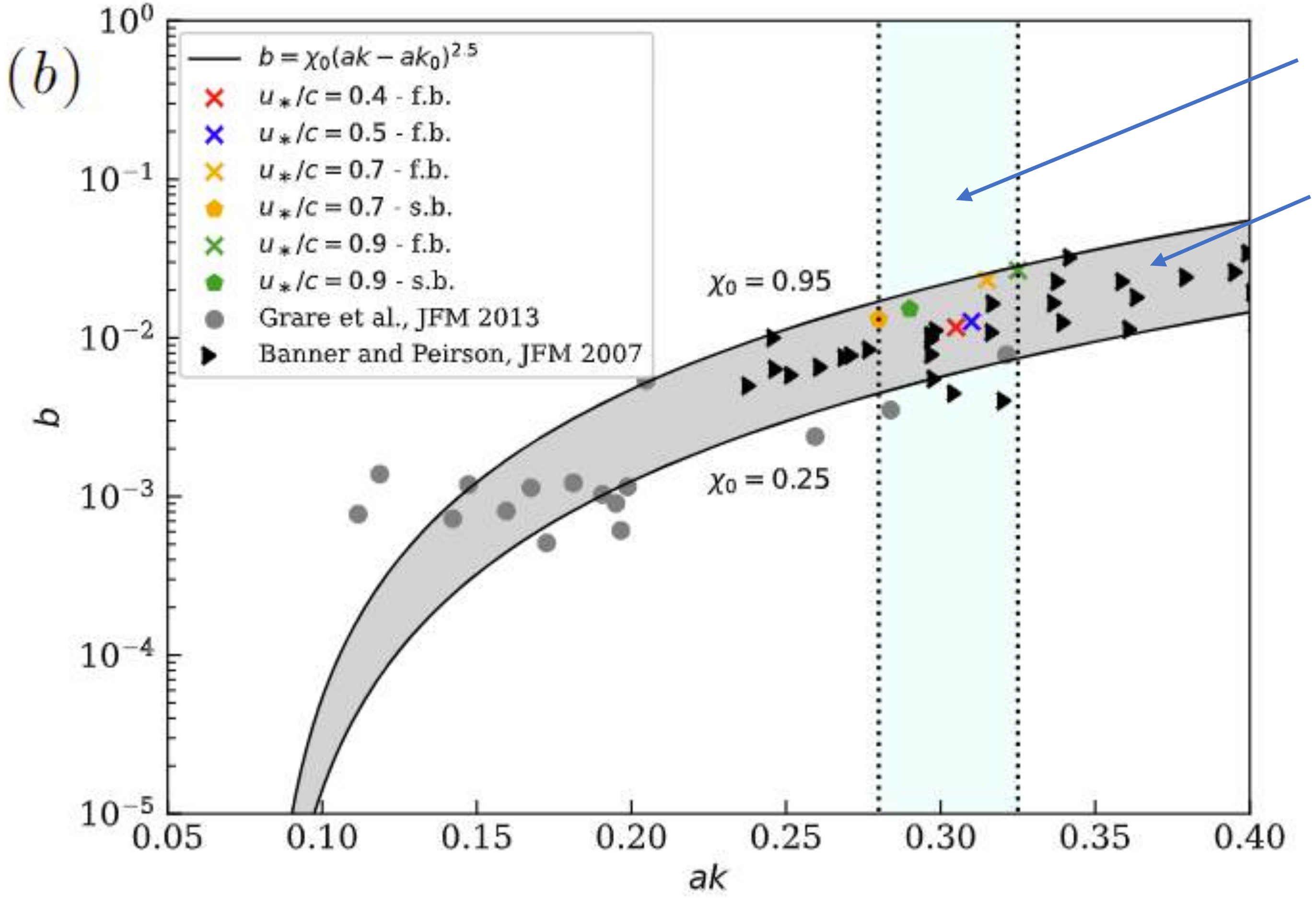
**Procedure:**

- Define the initial and the final points *(i)* and *(f)*.
- Compute the dissipation per unit of breaking crest $\varepsilon_l$;
- Compute **b**.
- Compare the **estimated b** with the one obtained from **the inertial scaling** $b = \chi_0(S - S_0)^{2.5}$.

# (3b) Energy dissipation (3/3)

**Drazen's inertial scaling:** $b = \chi_1 (S - S_0)^{2.5}$

$\chi_1 = O(1)$ and $S_0 = 0.08.$




**breaking interval for our simulations**

**Interval of** $\chi_1 \in [0.25 - 1]$

**Key points:**

- the **breaking event** appears to be a universal process, i.e. independent of the cause triggering the breaking;
- If choose a proper $w_c$ and $l_c$, we can predict the breaking-induced dissipation even for **wind-forced breaking waves**

# Conclusions

- **Direct numerical simulations** of wind-forced wave breaking at high wind speed;
- **Nonmonotonous behaviour** of the pressure force which reduces after the breaking stage (even without droplets);
- **Reduction is linked to the airflow modulation** and to the change in the velocity profile;
- **Growth-rate** and **energy dissipation** consistent with the available scaling laws.

N. Scapin et al., "*Momentum and energy fluxes in wind-forced breaking waves*" (to be submitted)

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# Numerical methodology

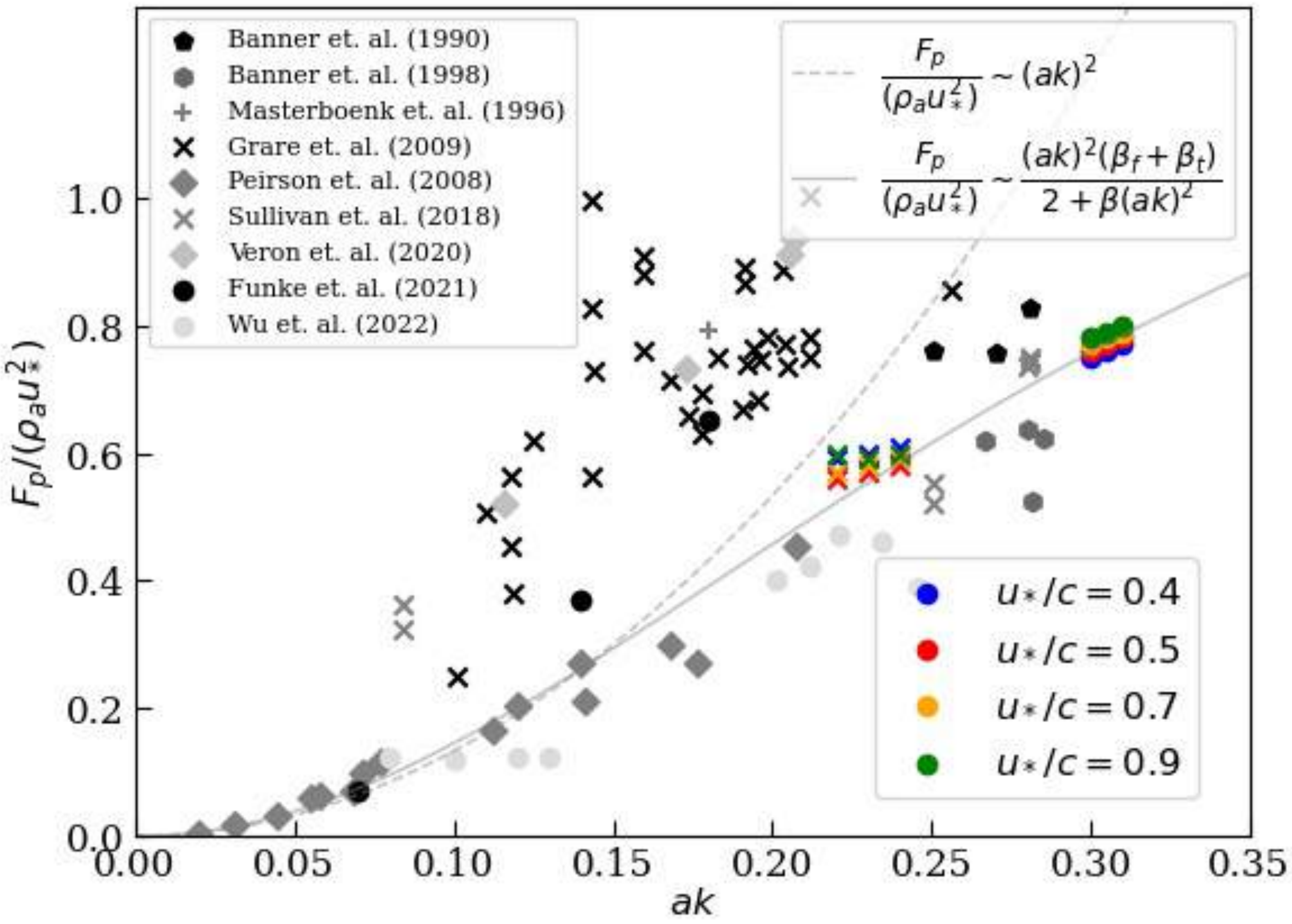
We solve on an adaptive cartesian grid (1) continuity equation (incompressibility constrain) together with (2) the momentum equation for a two-phase system.

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho \left( \partial_t \mathbf{u} + \nabla \cdot (\mathbf{u}\mathbf{u}) \right) = -\nabla p + \nabla \cdot \left[ \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \rho \mathbf{g} + \sigma \kappa \delta_\Gamma \mathbf{n}_\Gamma$$

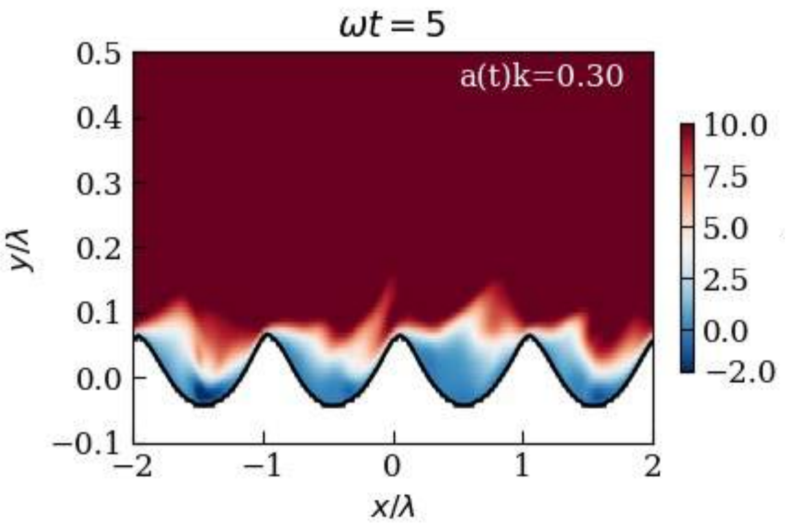
- Sharp-interface formulation for the interface advection (geometric VoF);
- Momentum consistent formulation to ensure robustness at high density ratio;
- Adaptive mesh-refinement (AMR) techniques based on wavelet transformation;
- **Basilisk:** Open-source implementation available at **http://basilisk.fr/**

# Comparison with the commonly employed scaling laws

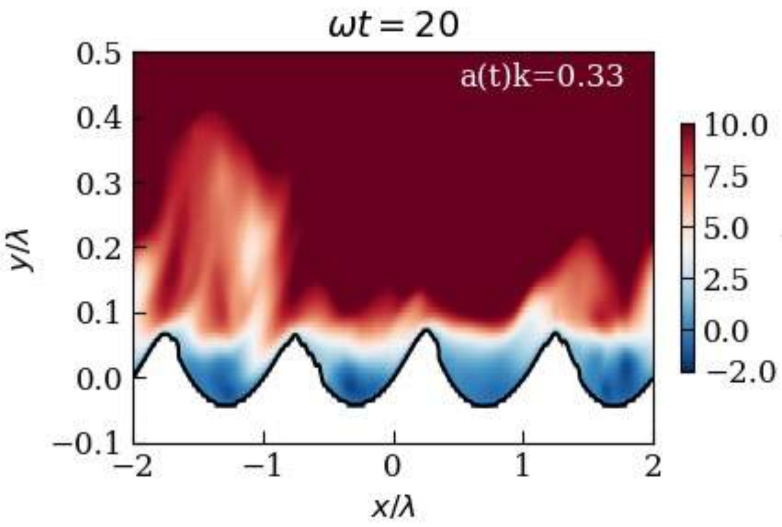



- **Pressure drag** vs the instantaneous steepness, before the breaking **(circles)** and after the breaking (**x**);
- Correction required at **larger steepness**, i.e. **a(t)k>0.15**

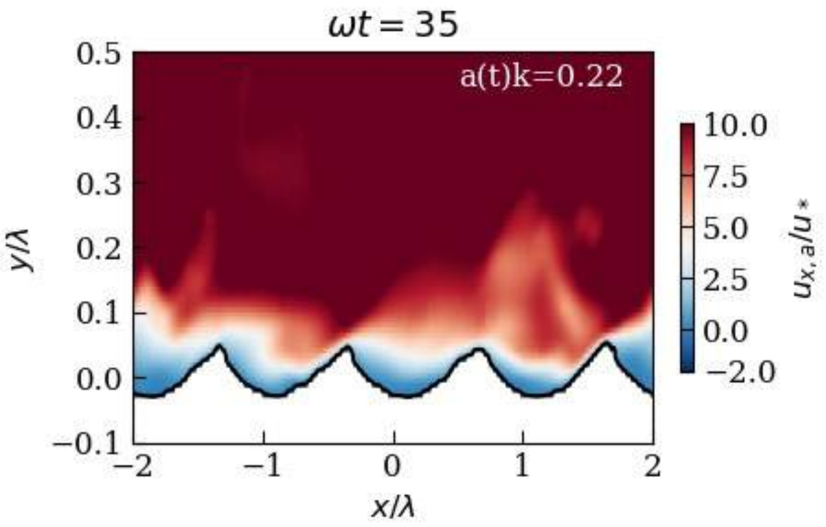
# 2. Airflow modulation




**Pre-breaking stage**

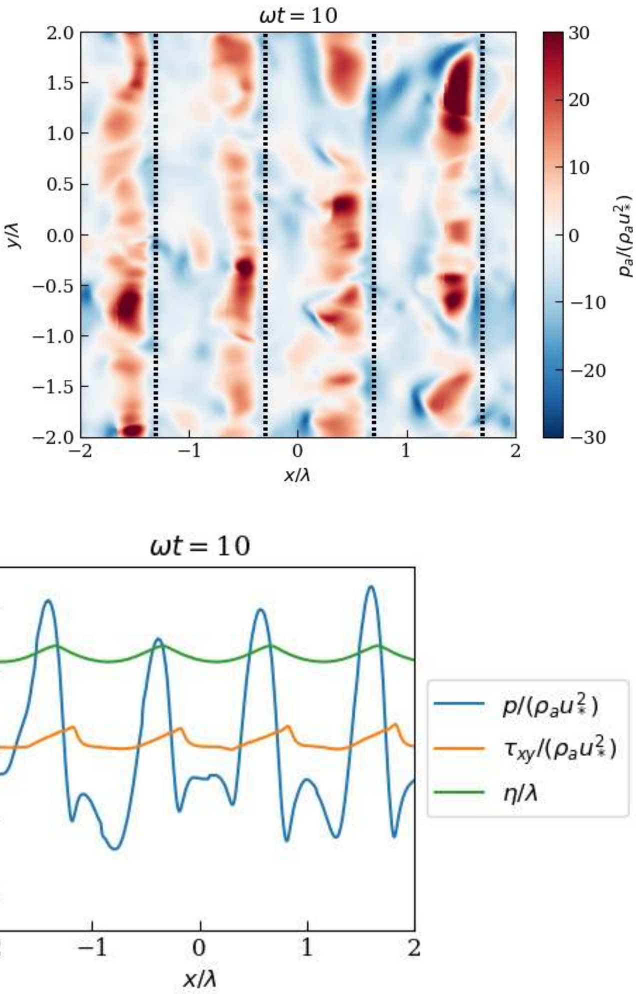



**Breaking stage**

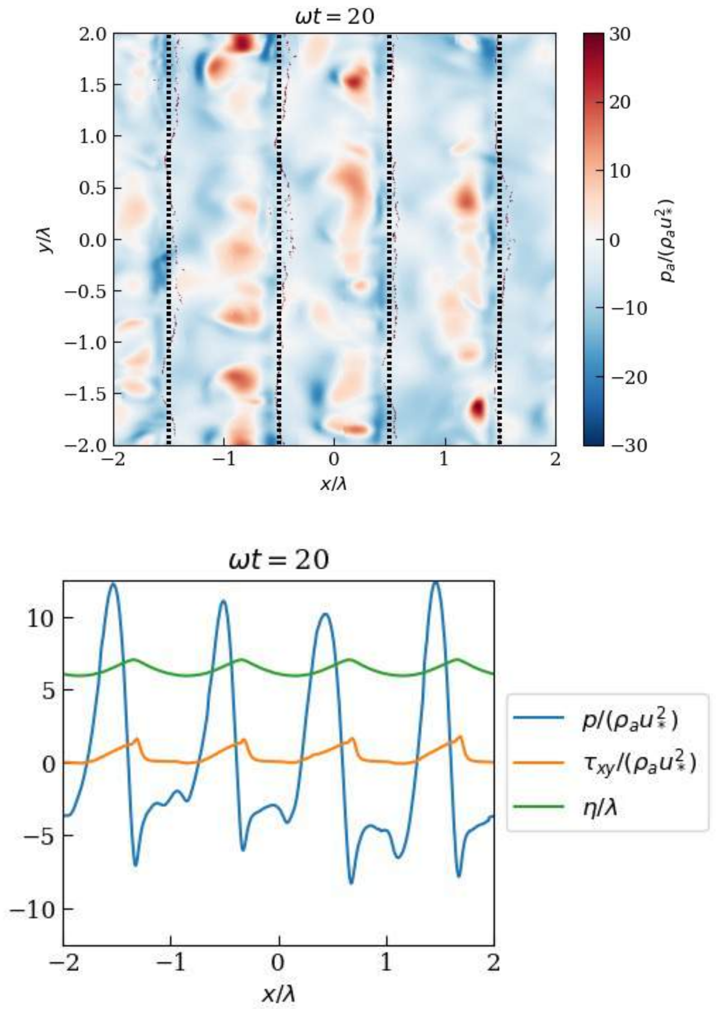



**Post-breaking stage**

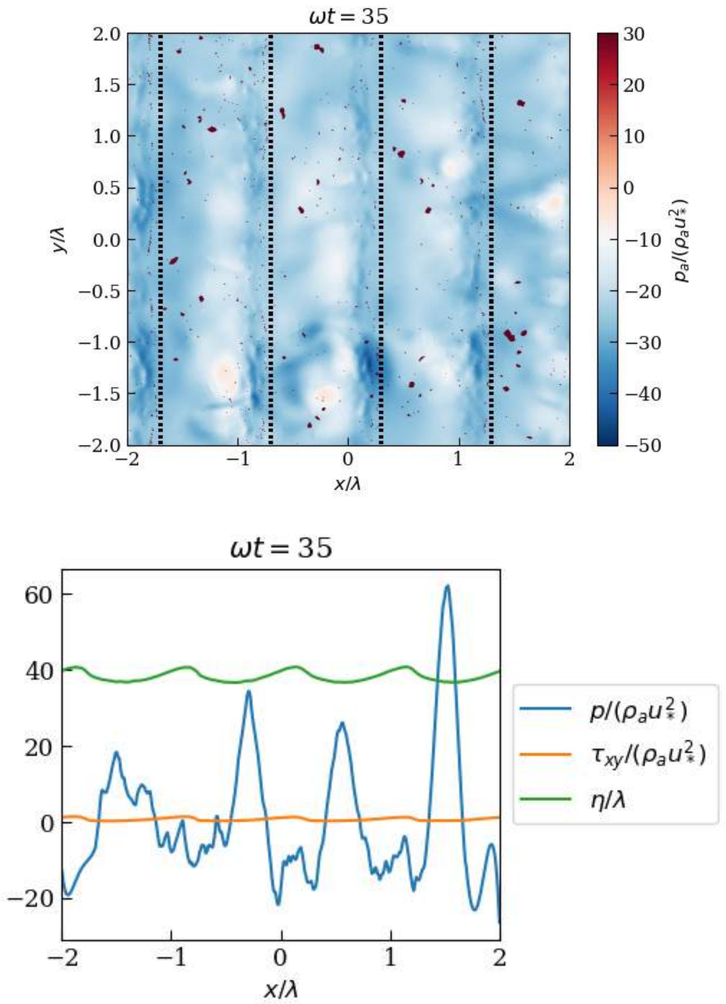
# Pressure distribution ($a_0k$ = 0.3 - $u_*/c$ = 0.9)




Pre-breaking, a(t)k=0.31



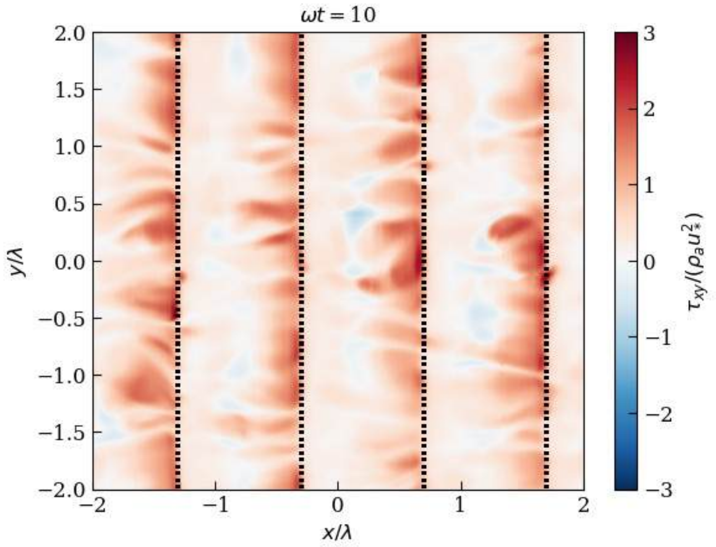

Breaking, a(t)k=0.33



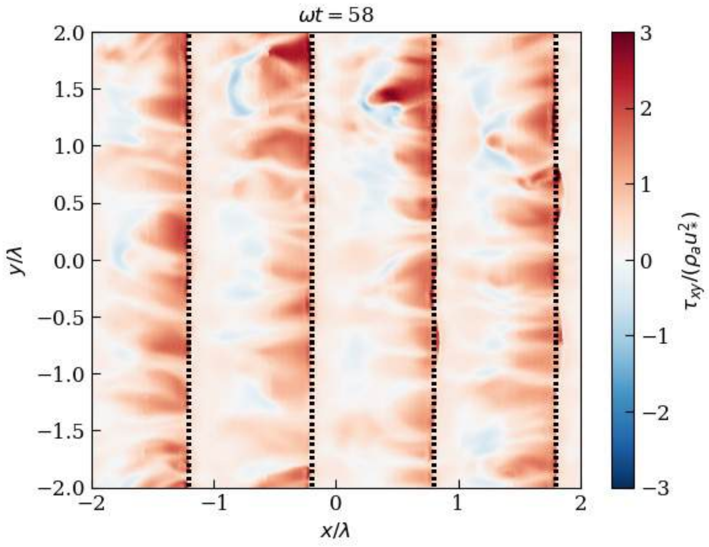

Post-Breaking, a(t)k=0.22

# Viscous stress distribution (a0k = 0.3 - $u_*/c$ = 0.5-0.9)

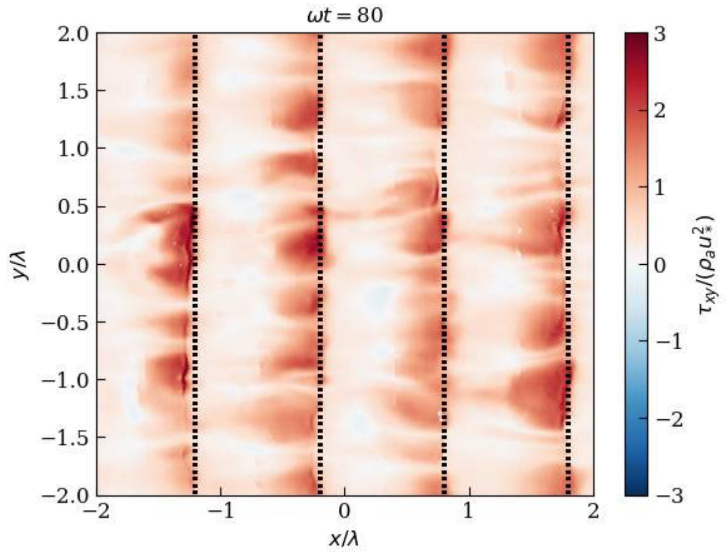
**$u_*/c$ = 0.5**




Pre-breaking, a(t)k=0.30

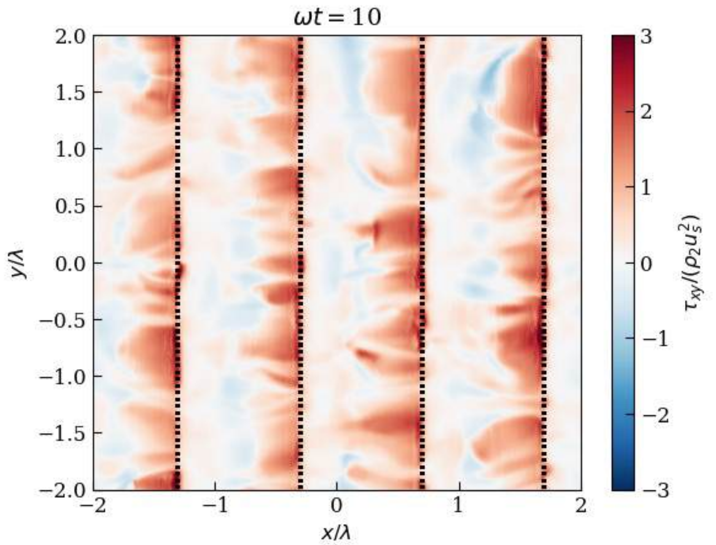



Breaking, a(t)k=0.32

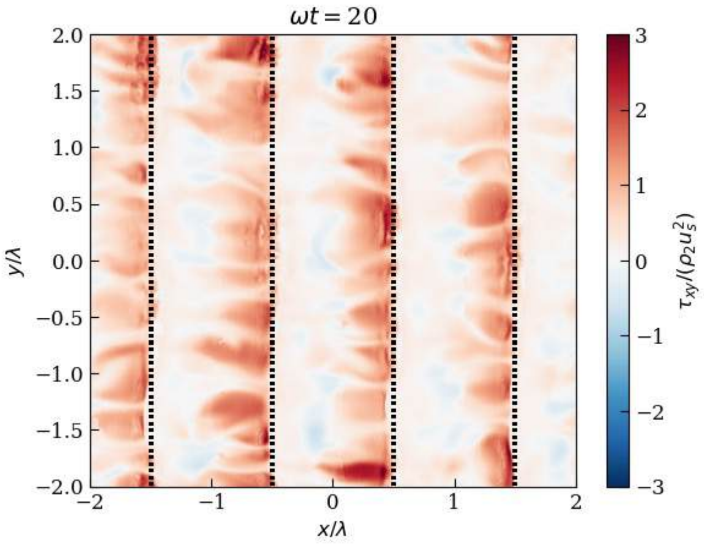



Post-Breaking, a(t)k=0.22

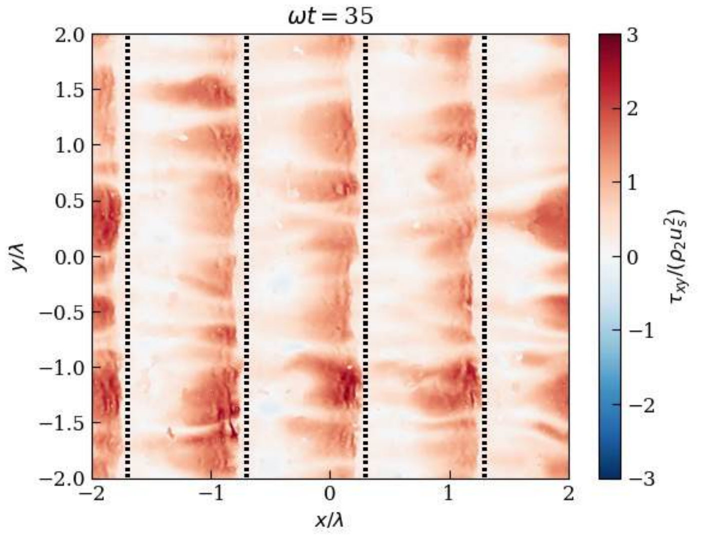
**$u_*/c$ = 0.9**




Pre-breaking, a(t)k=0.305

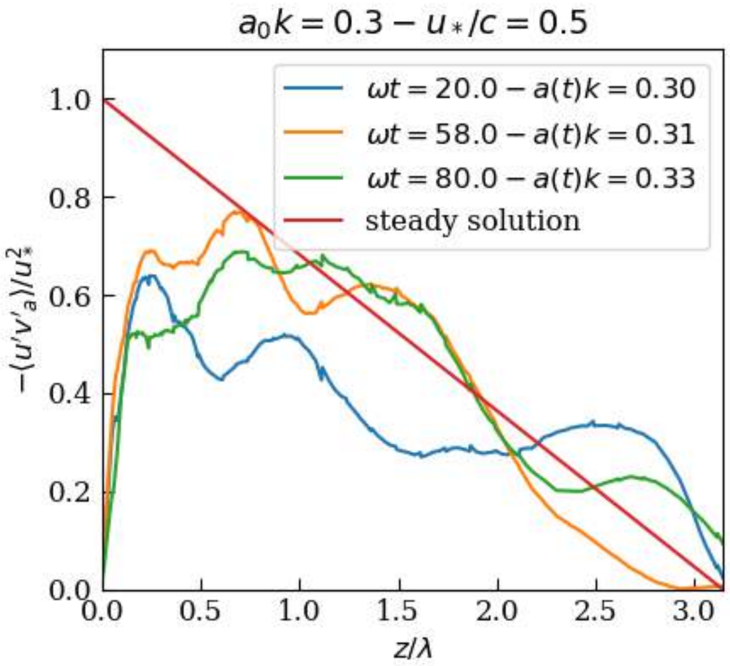


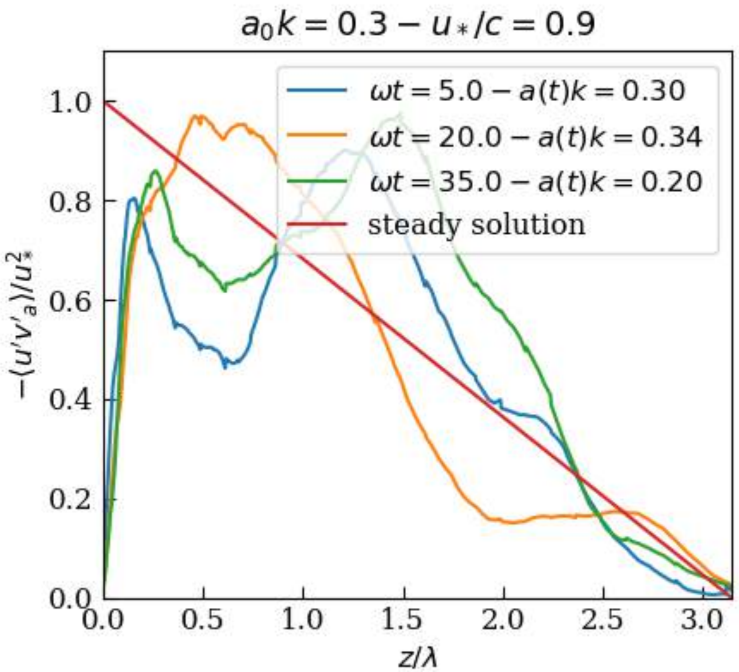

Breaking, a(t)k=0.33




Post-Breaking, a(t)k=0.21

# Reynolds stress




Large variation of the Reynolds stress in time with an enhancement at the breaking stage
(which also explain the deviation from the log-law)