High Meadows Environmental Institute

MECHANICAL & AEROSPACE ENGINEERING

Growth rate and energy dissipation in breaking waves at high wind speeds

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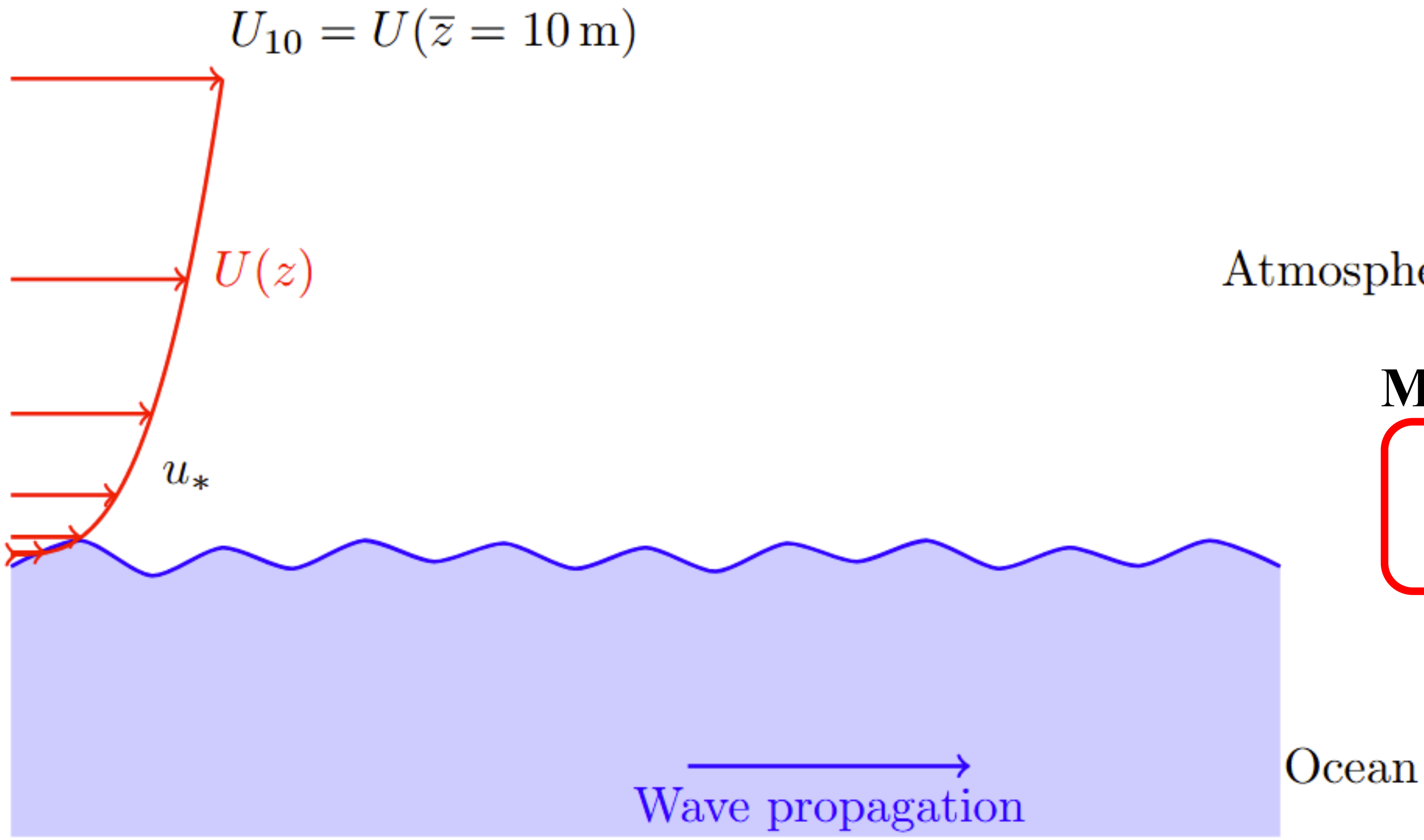
Wind-forced breaking waves

Wind-forced wave breaking
modulate the exchanges of
momentum, **energy** and mass at the ocean-atmosphere interface

Energy exchanges at the ocean-atmosphere



Atmosphere → **Turbulent airflow**

↓

Momentum fluxes

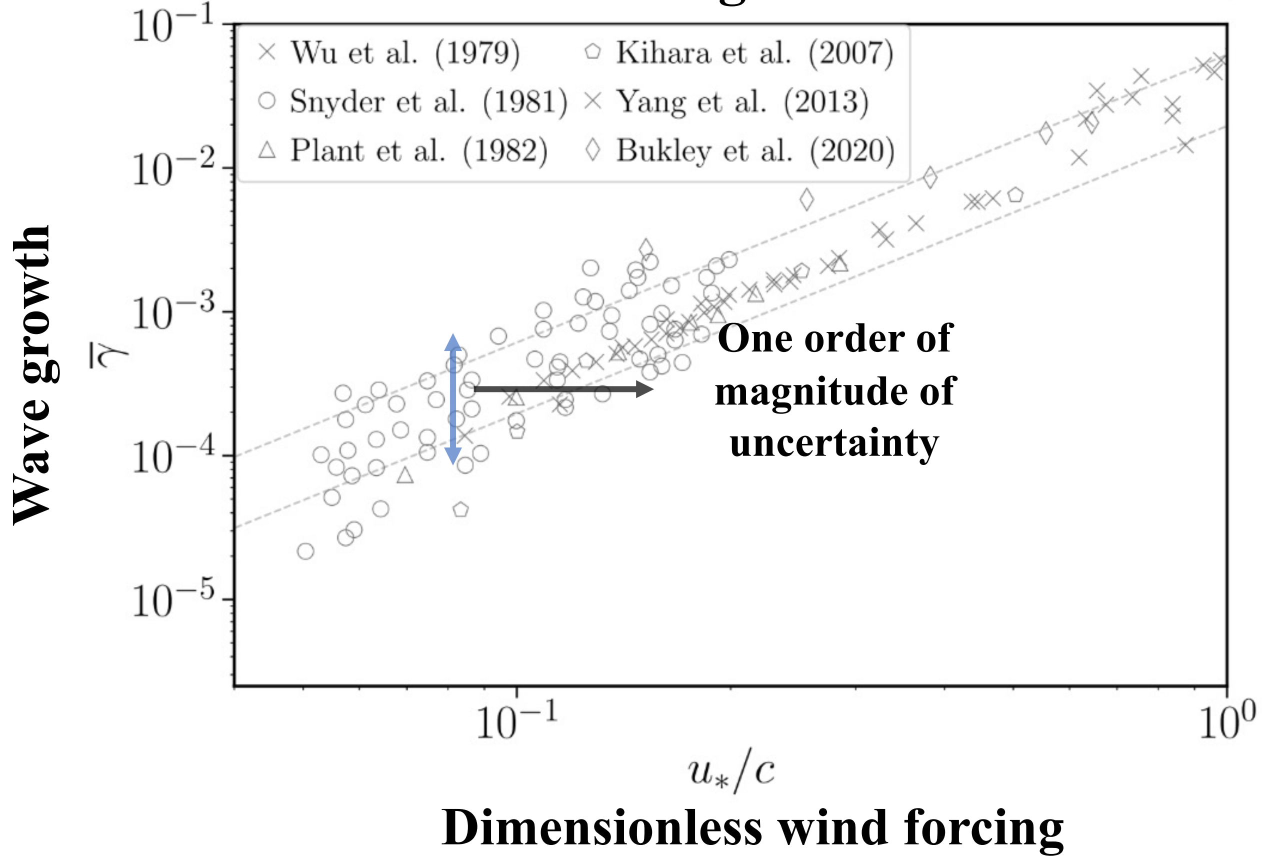
Energy fluxes/ Growth rate

← **Waves and underwater currents**

Dissipation

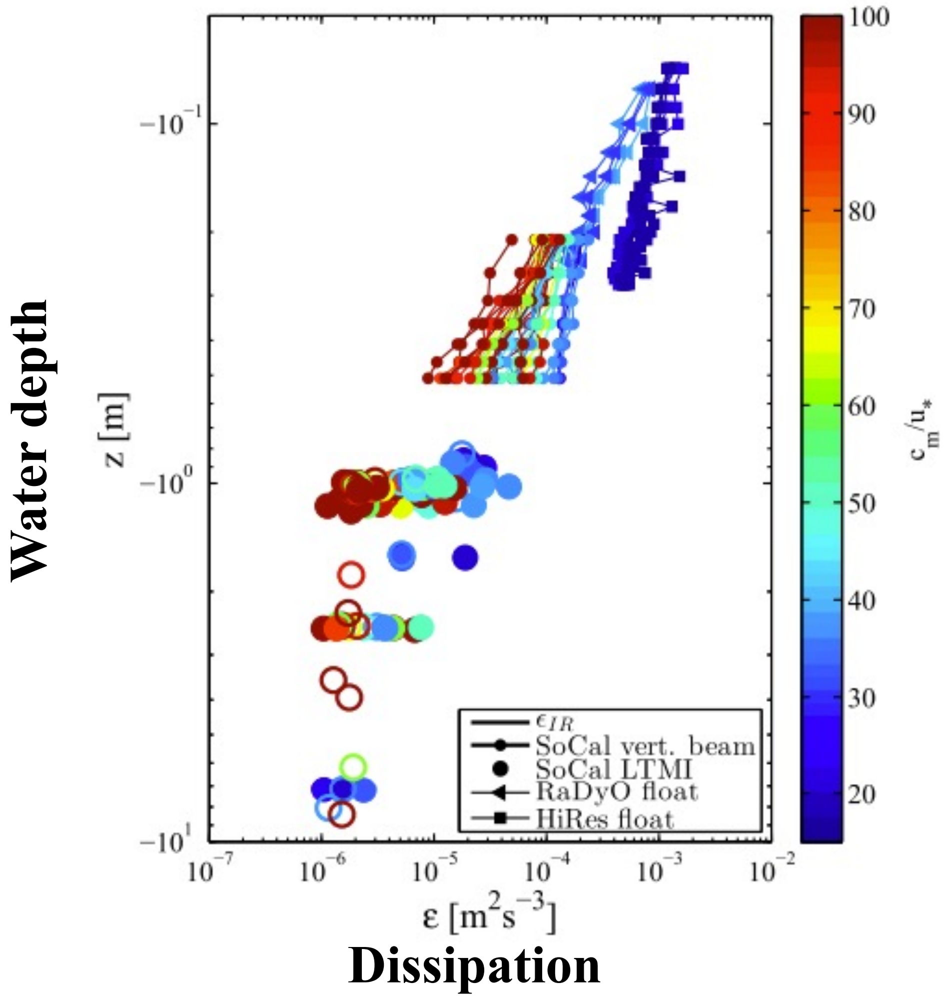
Wave growth and energy dissipation

Wave growth



For the same value of u_*/c, we get very different values of $\bar{\gamma}$.

Energy dissipation

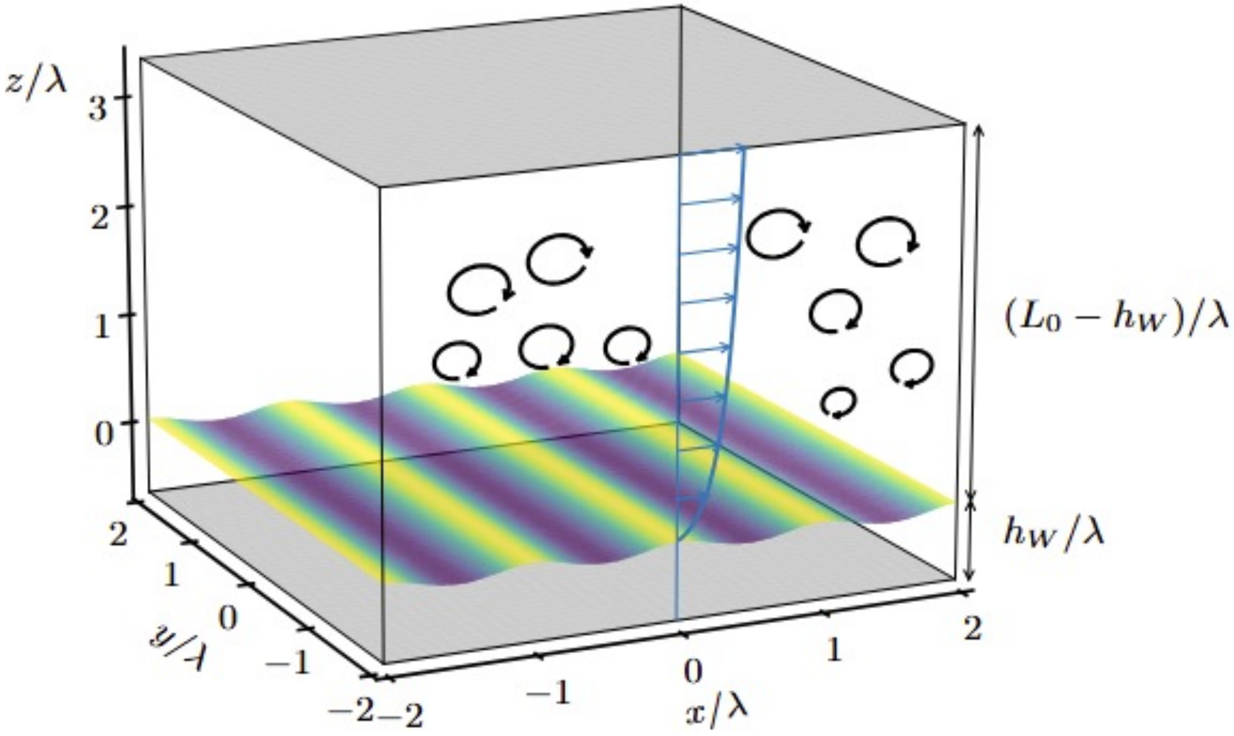


Large scatter up to one order of magnitude

Can we explain this variability?

Configuration set-up

- **Initial condition in Air**: fully-developed turbulence
- **Initial condition in Water**: potential flow solution of a third-order Stokes wave.



Computational domain:

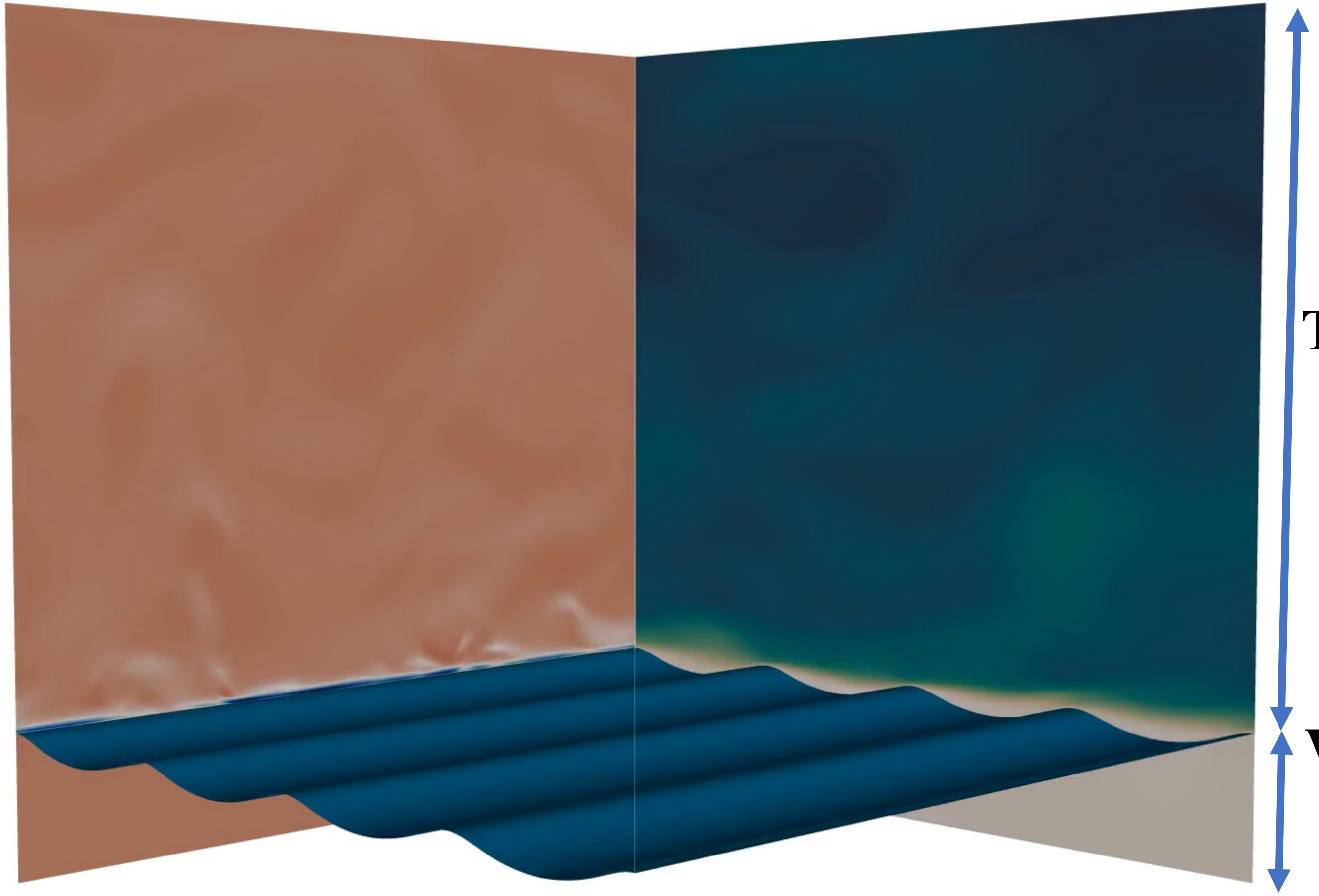
- $4\lambda \times 4\lambda \times 4\lambda$, $h_w \approx 0.64\lambda, L_0 - h_w \approx 3.36\lambda$
- x-y: periodic directions; z: free-slip conditions;
- Grid resolution: $L^{10} - L^{11}$(i.e. $1024^3 - 2048^3$);

We fix:
$Re_* = 720, Re_w = 2.5 \cdot 10^4, Bo = 200, a_0 k = 0.3$

We vary (in the **high-wind speed** regime):

$$\frac{u_*}{c} = 0.3 - 0.4 - 0.5 - 0.7 - 0.9;$$

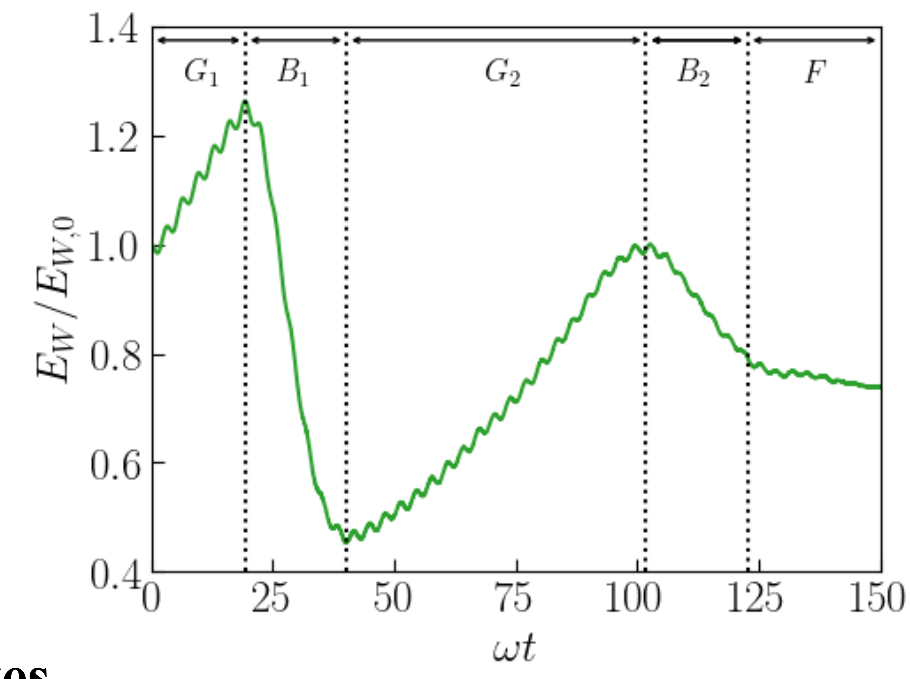
Fully-resolved direct numerical simulations
using **Basilisk solver (http://basilisk.fr/)**



Turbulent airflow

Water waves and water currents

Wave energy curve



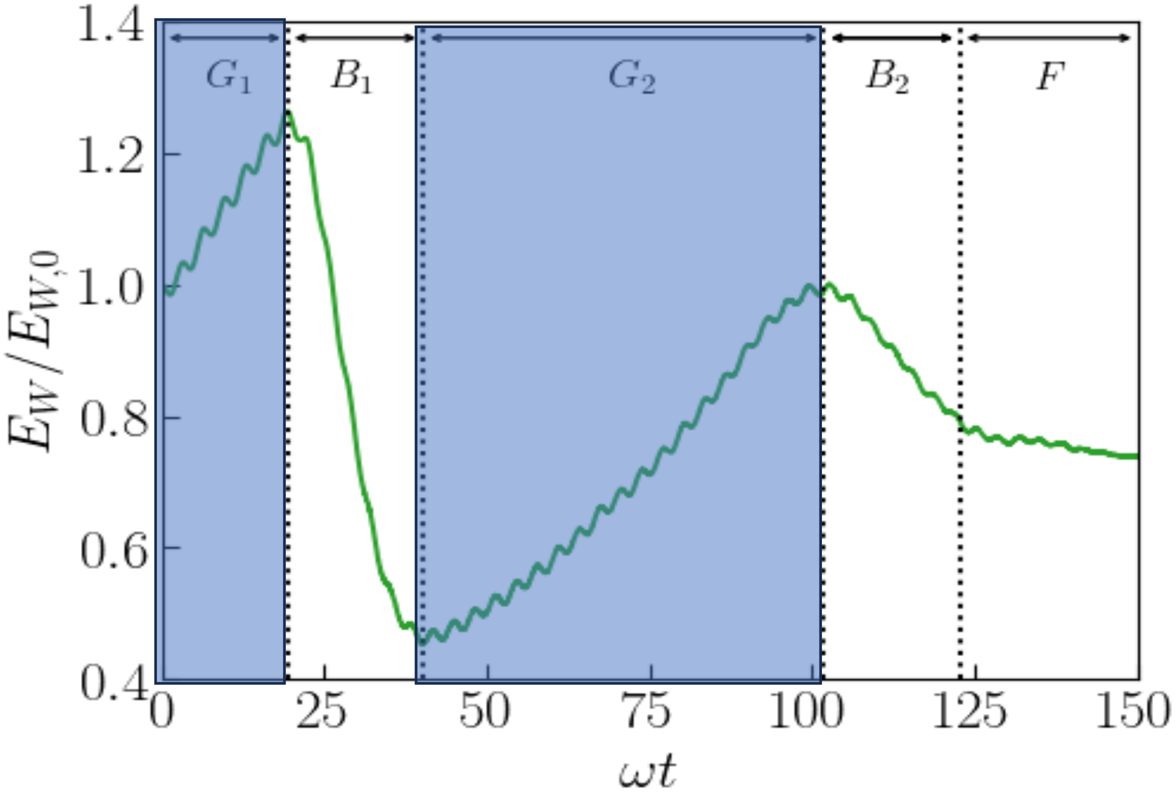
$$E_w(t) = \rho_w |\boldsymbol{g}| \int_{\Omega_w} (z - z_0) dV$$

t = 0.000

<u>The instantaneous change in $E_W(t)$, i.e. the change in the wave steepness</u>

- Affects **the momentum fluxes (Scapin 2025, JFM), the wind input**, i.e. S_{in}, and **the growth rate**
- During growing, $E_W(t)$ is stored in the waves, during breaking, $E_W(t)$ is **dissipated** in the water column

Wave energy curve



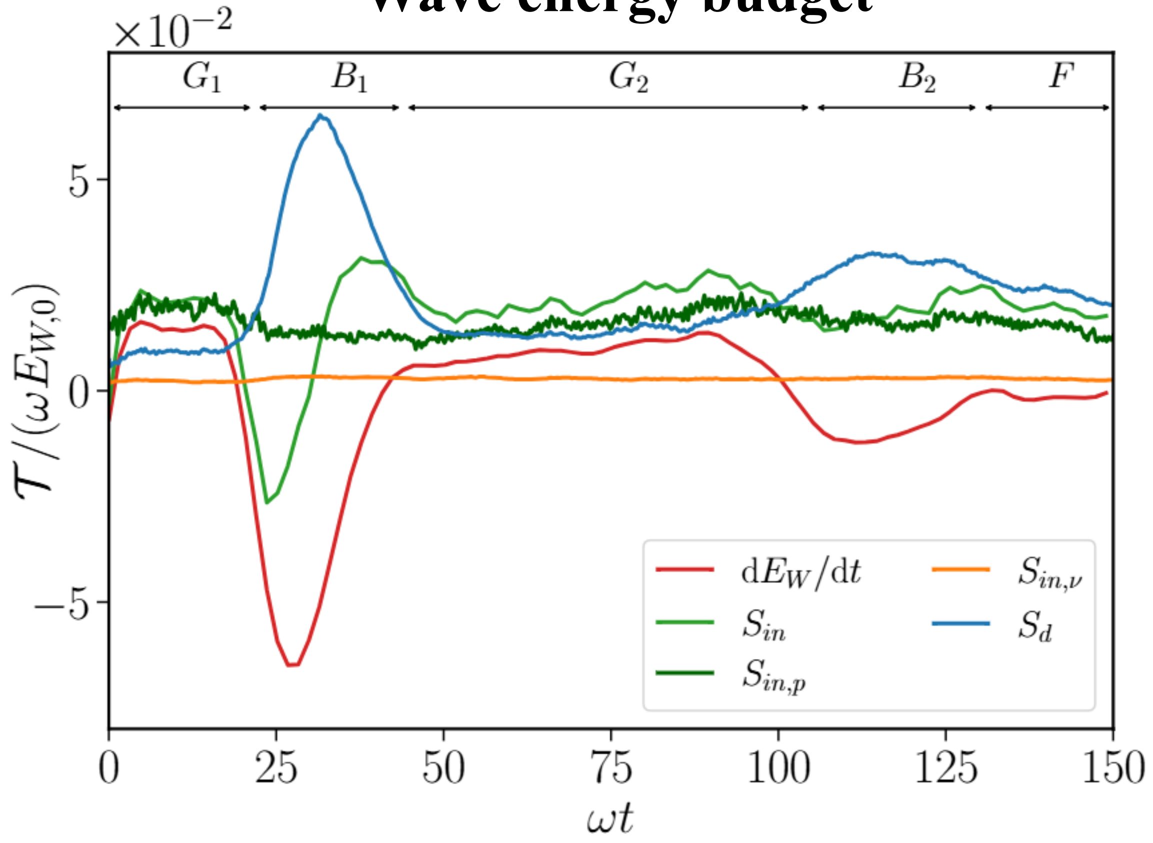
$$E_w(t) = \rho_w |\boldsymbol{g}| \int_{\Omega_w} (z - z_0) dV$$

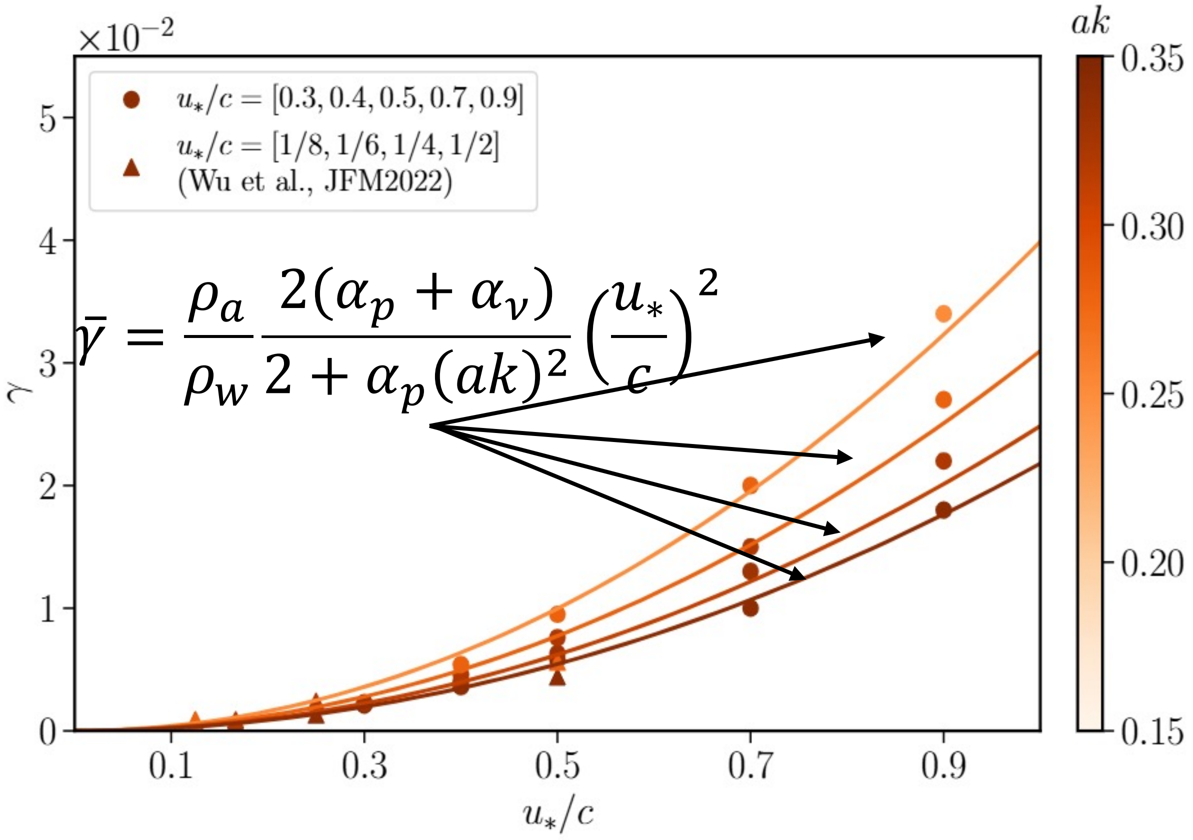
1: Wave growth

2: Dissipation during breaking

Wave growth (1/2)

Wave energy budget





$$\bar{\gamma} = \frac{\rho_a}{\rho_w}\frac{2(\alpha_p + \alpha_\nu)}{2 + \alpha_p (ak)^2}\left(\frac{u_*}{c}\right)^2$$

$$\frac{dE_w}{dt} = S_{in} - D \rightarrow D = -\frac{\mu_w}{2}\int_{\Omega_w}\left(\partial_i u_j + \partial_j u_i\right)^2 dV$$

$$\rightarrow S_{in} = -\int_{\Gamma} (p\boldsymbol{I} + \boldsymbol{\tau}) \cdot \boldsymbol{n} \cdot \boldsymbol{u}\, dS$$

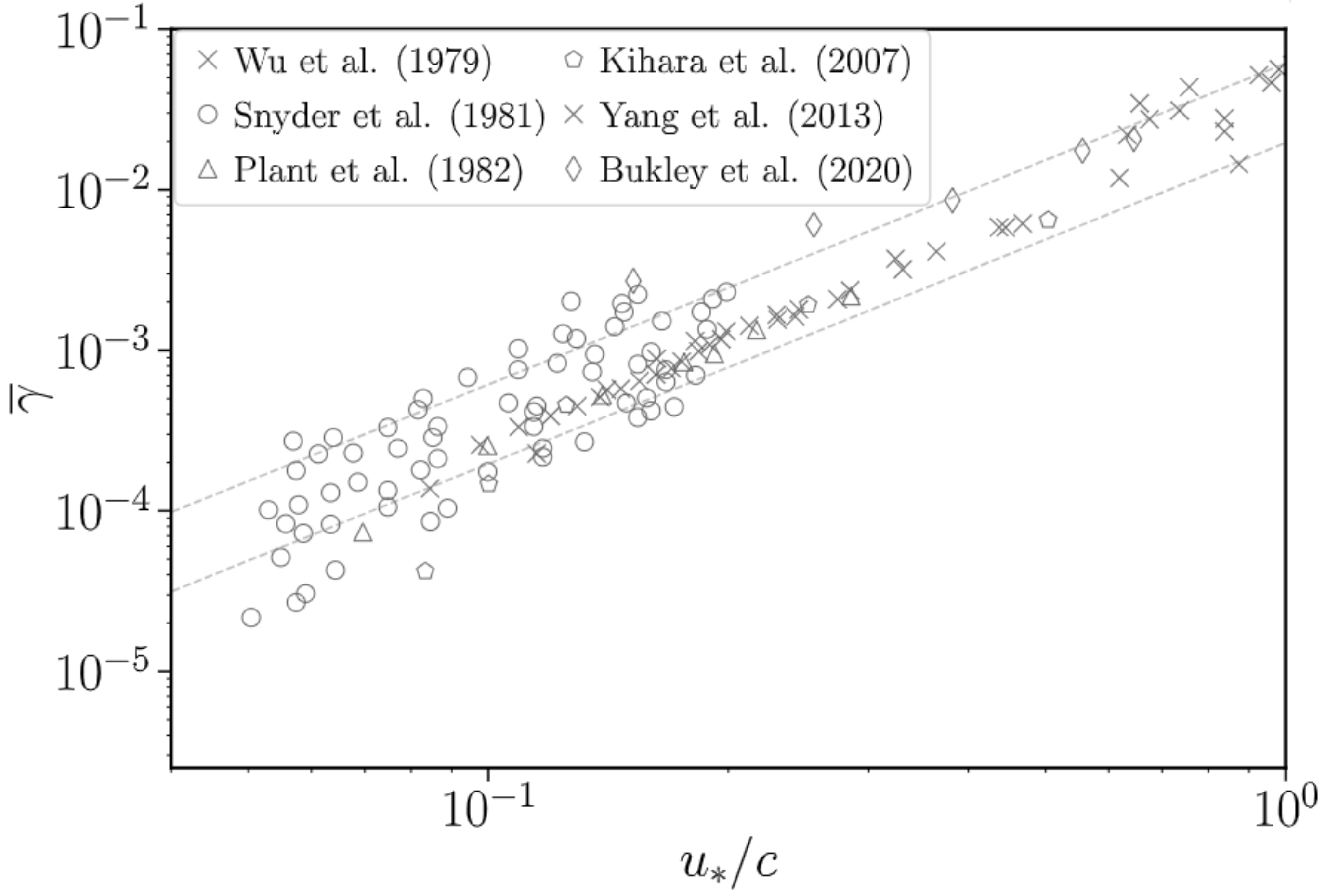
We extract the growth rate $\gamma(t) = \frac{S_{in}(t)}{\omega E_w(t)}$

↓

Strongly modulated from the steepness

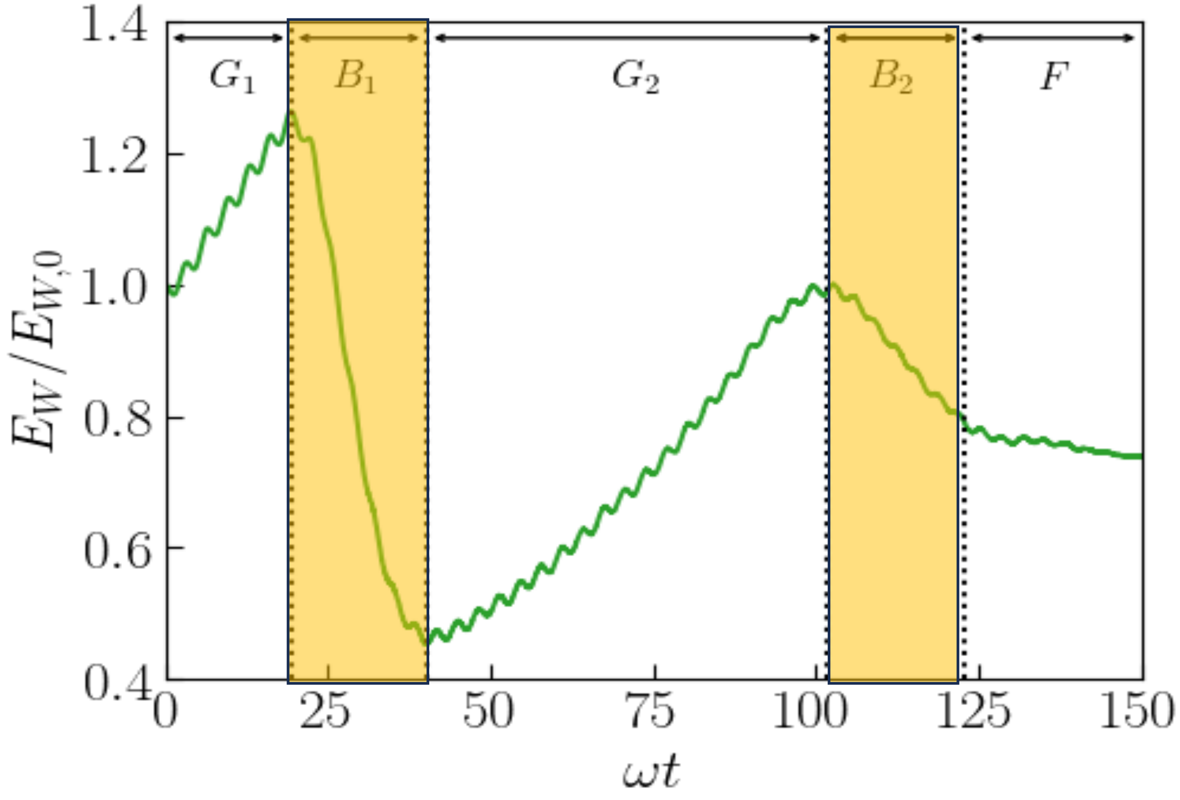
Belcher & Hunt (JFM, 1993)

Wave growth (2/2)



A key source of variability: wave steepness

Wave energy curve

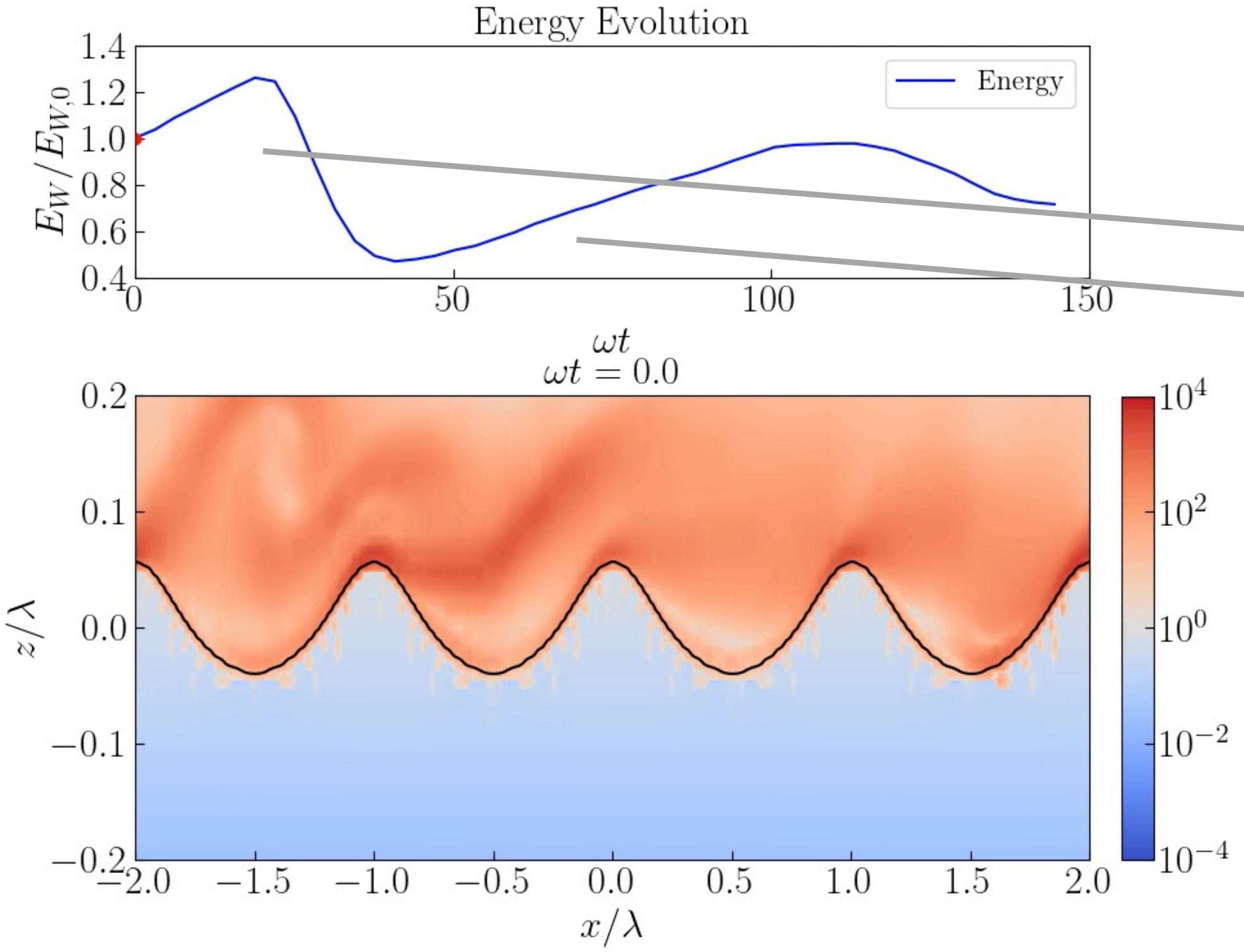


$$E_w(t) = \rho_w|\boldsymbol{g}|\int_{\Omega_w}(z - z_0)dV$$

1: Wave growth

2: Dissipation during breaking

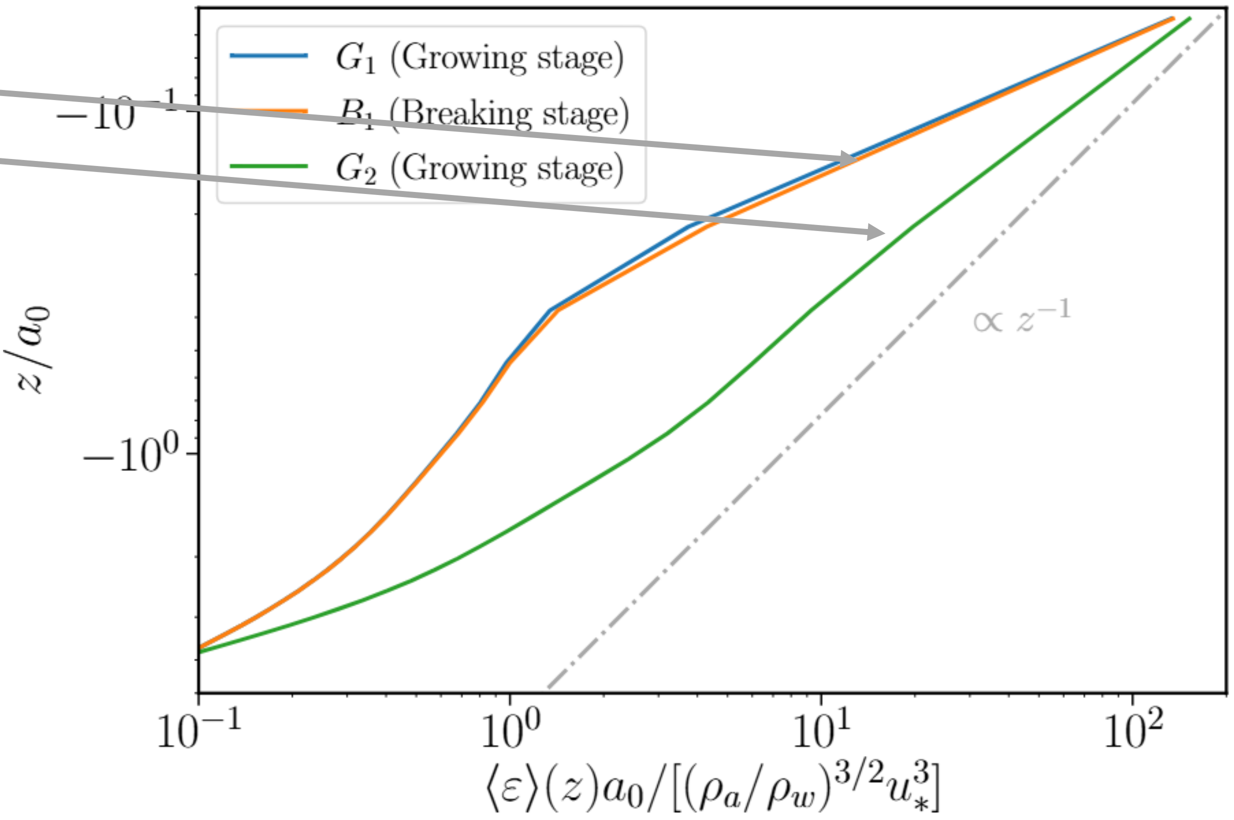
Wave breaking-induced dissipation



Dissipation negligible during G_1

Dissipation starts to become larger during B_1 and is transported in the water column during B_2

Profile during G_1, B_1 and G_2

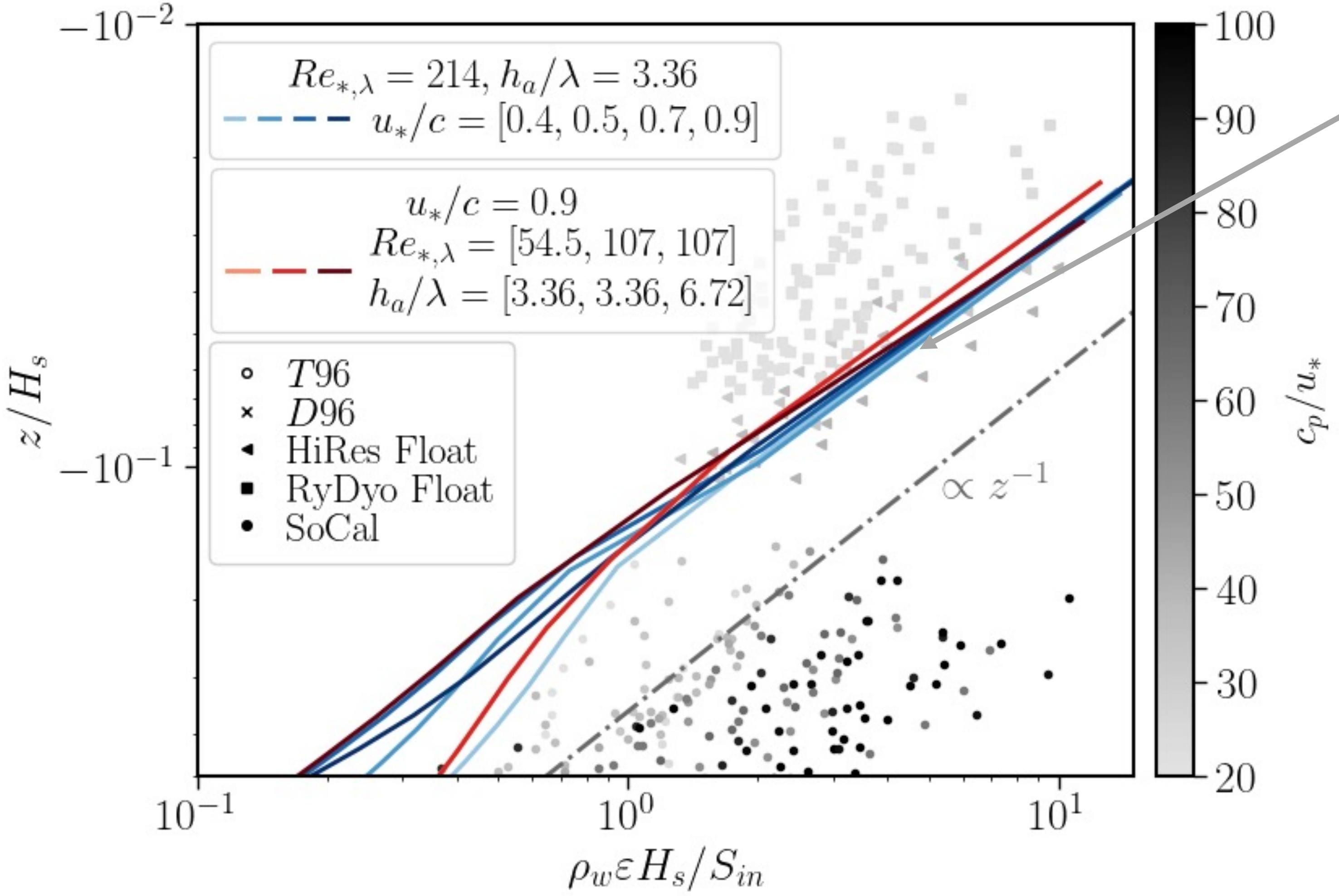


Wave breaking promotes the transition of the dissipation profile!

Scaling the underwater energy dissipation (1/3)

Sutherland and Melville (JPO, 2015) proposed to rescale ε as

$$\frac{\rho_w \varepsilon(z) H_s}{S_{in}} = f\left(\frac{z}{H_s}\right)^{-1} \qquad S_{in} = F_{p,x} c \approx \rho_a u_*^2 c$$



- **A very good collapse** of the dissipation profiles within $0.1H_s$
- **Numerical results are compatible with the field data** at the lowest wave age near the surface

Scaling the underwater energy dissipation (2/3)

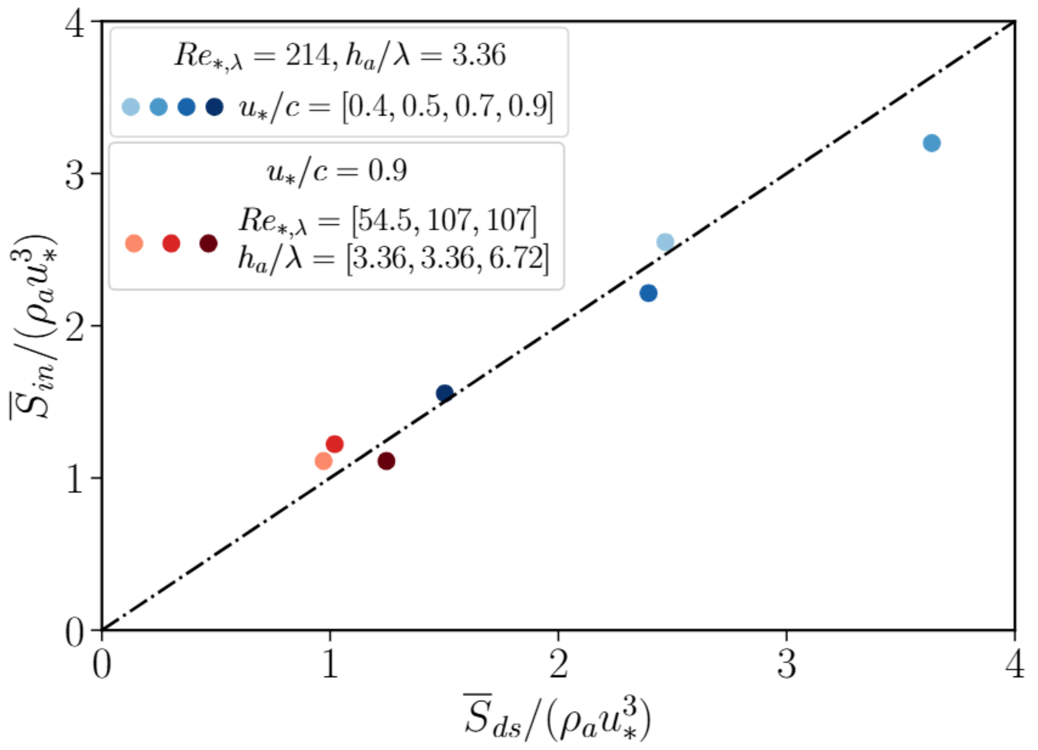
From the scaling proposed in Sutherland and Melville (JPO, 2015), we can derive

$$\frac{\rho_w \varepsilon(z) H_s}{\bar{S}_{in}} = A\frac{H_s}{z} \rightarrow \varepsilon(z) \sim \frac{u_*^2 c}{z}$$

Balanced between wind input and energy dissipation

$$\frac{\rho_w \varepsilon(z) H_s}{\bar{S}_{in}} = A\frac{H_s}{z} \longrightarrow \underbrace{\int_{-h_0}^{\eta} \varepsilon(z) dz}_{\bar{S}_{ds}} = \bar{S}_{in}\frac{A}{\rho_w}\log\left(\frac{\eta}{h_0}\right)$$

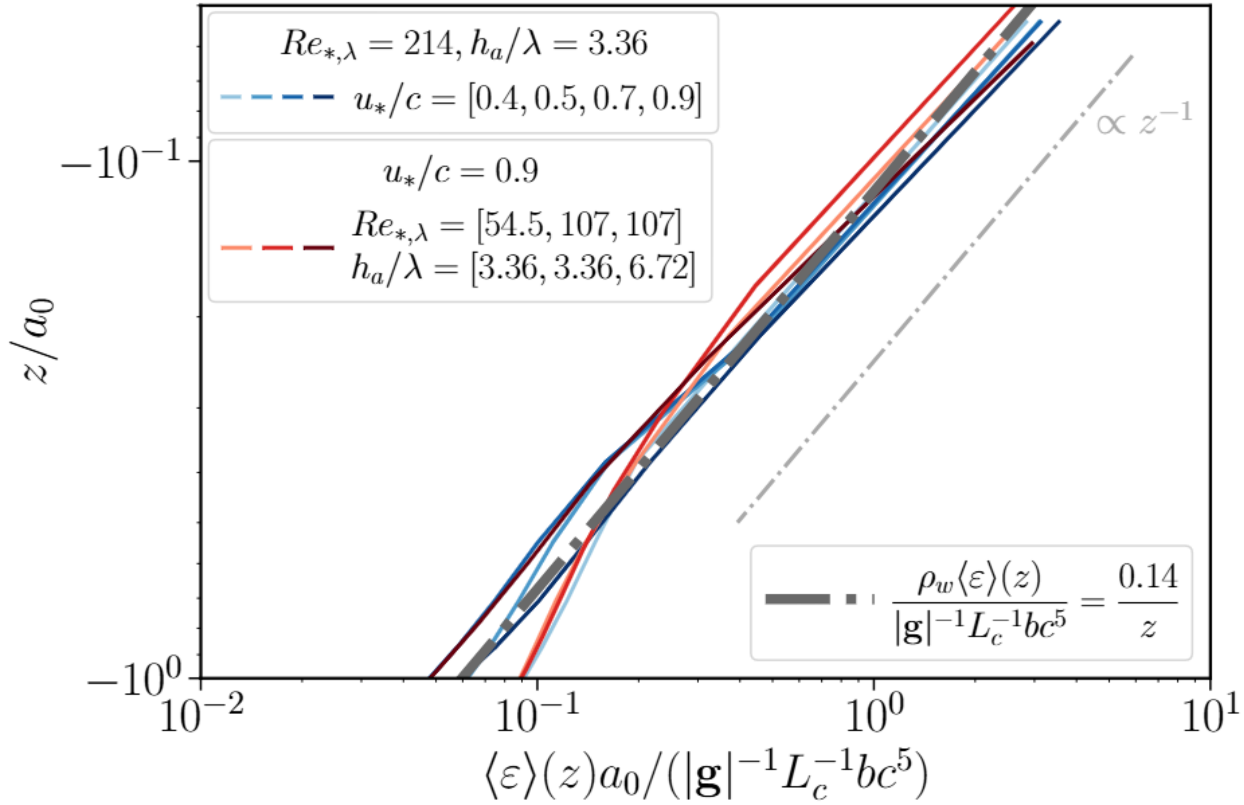
$$\boldsymbol{\bar{S}_{in} \sim \bar{S}_{ds}}$$



Scaling the underwater energy dissipation (3/3)

$$S_{in} \sim S_{ds} = \frac{\rho_w}{g}\int b\Lambda(c)c^5 dc = \frac{\rho_w}{g}\frac{bc^5}{L_c}$$

Philips (JFM 1985)



- **A very good collapse** of the dissipation profiles within $0.1H_s$
- **Given** $S_{in} \sim S_{ds}$, this **dissipation-based scaling** is fully consistent with the **wind-input based scaling**.

Consistent with our understanding of wave breaking:

wave breaking occurs when fluid inertia overcome restoring forces. The cause, e.g. wind, sets the onset of breaking. Once breaking starts, the energy loss is independent from the cause

Conclusions

Direct numerical simulations of wind-forced wave breaking at high wind speed to extract the wave growth rate and breaking-induced dissipation

Wave growth rate

- **Wave growth scales as $(u_*/c)^2$ under wind forcing, with strong modulation due to wave slope** following **non-separated sheltering**

Breaking-induced dissipation

- **Wave breaking is sufficient to promote the transition of ε** to $\sim z^{-1}$
- **New scaling law** to unify the **dissipation profile across different u_*/c**

N. Scapin et al., "*Growth and dissipation in wind-forced breaking waves*", Geophysical Research Letters

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Numerical methodology

Direct solution of *(1)* continuity equation (incompressibility constraint) with *(2)* the momentum equation for a **two-phase system**

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\rho\left(\partial_t \boldsymbol{u} + \nabla \cdot (\boldsymbol{u}\boldsymbol{u})\right) = -\nabla p + \nabla \cdot (\mu(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)) + \sigma\kappa\delta_\Gamma + \rho \boldsymbol{g}$$

Main features of the numerical algorithm:

- **Sharp-interface formulation** for the interface advection (geometric VoF)
- **Momentum consistent formulation** to ensure robustness at high density ratio
- **Well-balanced formulation** to avoid artificial parasitic currents at the interface
- **Adaptive mesh-refinement (AMR)** techniques based on wavelet transformation

Basilisk: Open-source implementation available at **http://basilisk.fr/**

Wave breaking-induced dissipation

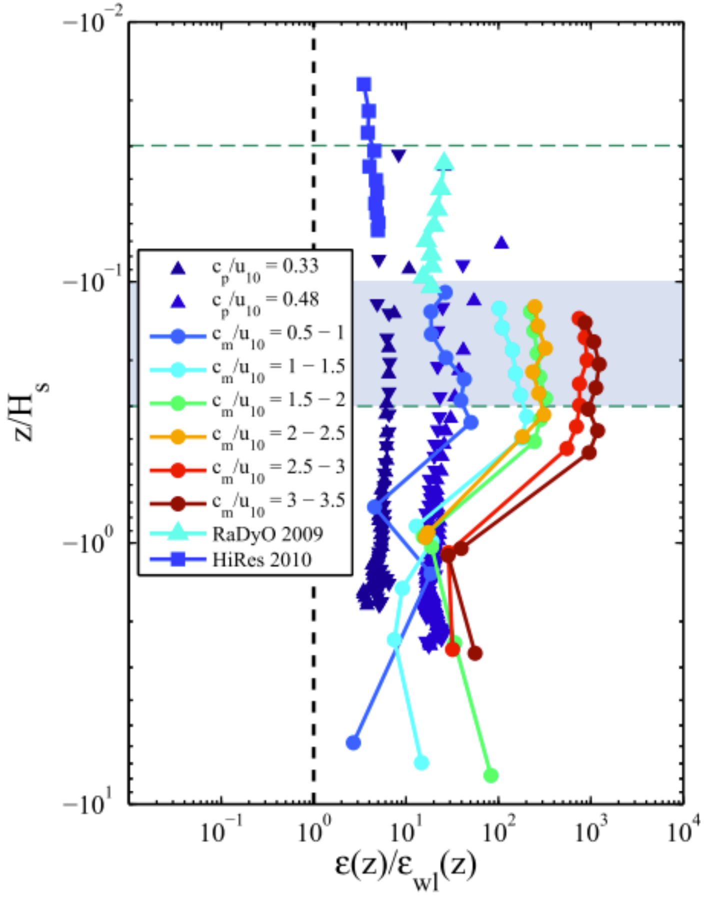
The wall-layer scaling argument is an incorrect scaling for the turbulent dissipation

Wall-layer scaling

$$\varepsilon_{wl}(z) = \frac{u_*^3(\rho_a/\rho_w)^{0.5}}{\kappa}\frac{u_*^3}{z}$$

Present scaling

$$\frac{\rho_w \varepsilon(z) H_s}{\bar{S}_{in}} = A\frac{H_s}{z} \rightarrow \varepsilon(z) \sim \frac{u_*^2 c}{z}$$



Wave growth: Belcher's scaling

We compute the wave growth rate

$$\gamma(t) = \frac{S_{in}(t)}{\omega E_w(t)}$$

Wind input: $S_{in} = -(p\boldsymbol{I} + \boldsymbol{\tau}) \cdot \boldsymbol{n} \cdot \boldsymbol{u}$

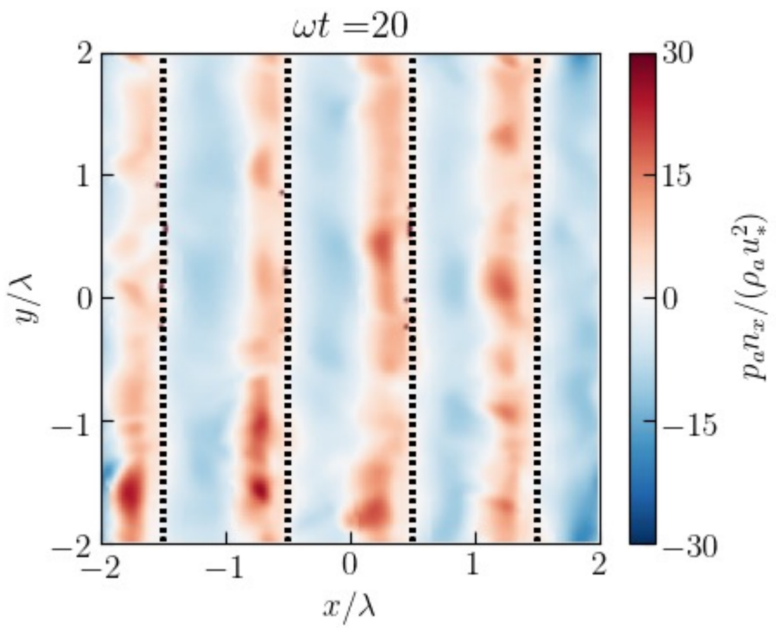
Wave energy $E_W = \rho_w g a^2(t)/2$

$S_{in} = \alpha_p (ak)^2 / 2\rho_a u_*^2 c$

$$\bar{\gamma} = \alpha_p \frac{\rho_a}{\rho_w} \left(\frac{u_*}{c}\right)^2$$

Miles's theory for the growth rate, γ

Belcher and Hunt (JFM, 1993) and Belcher (EJFM, 1999) proposes introduces the wave sheltering, i.e. an **asymmetric pressure distribution** due to the distortion of the airflow by the waves



Wind input due to pressure

$$S_{in,p} = \frac{\alpha_p (ak)^2}{2+\alpha_p (ak)^2} \rho_a u_*^2 c$$

Wind input due to viscous stress

$$S_{in,v} = \frac{\alpha_v (ak)^2}{2+\alpha_p (ak)^2} \rho_w u_*^2 c$$

$$\bar{\gamma} = \frac{\rho_a}{\rho_w} \frac{2(\alpha_p + \alpha_v)}{2 + \alpha_p (ak)^2} \left(\frac{u_*}{c}\right)^2$$

The growth rate is now corrected for the wave slope